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SHAKEDOWN SEISMIC ANALYSIS OF COMPOSITE STEEL CONCRETE SPATIAL FRAME

ОПТИМИЗАЦИЯ ПРЕДЕЛЬНОГО СОСТОЯНИЯ ПРОСТРАНСТВЕННОЙ СТАЛЕЖЕЛЕЗОБЕТОННОЙ РАМЫ ПРИ СЕЙСМИЧЕСКОМ ВОЗДЕЙСТВИИ

ABSTRACT

The earthquake analysis of buildings and structures can be performed by finding solution of the optimization problem, which takes into account the nonlinear properties of materials. There are several methods for the earthquake analysis using for design of the buildings, such as the analysis using the elastic response spectrum, non-linear static analysis, non-linear direct integration method using accelerograms of occurring earthquakes or artificial accelerograms and other [19]. The earthquake analysis of buildings and structures can be performed by finding solution of the optimization problem of shakedown analysis taking into account the nonlinear properties of materials. Such analysis has some advantages. External actions are introduced as the set of the load cases, that's why we can solve the problem for all direction of seismic load and for every scheme of live load at once. As part of the solution of the optimization problem we can take into account the nonlinear behavior of the elements.

This paper presents new solution for analysis of composite steel-concrete frame buildings with elastic-plastic and brittle elements. We accepted concrete and composite elements behavior exposed to shear force as brittle behavior. On other side, we accepted elements behavior exposed to bending moment as elastic-plastic behavior.

It is assumed that the load varies randomly within the specified limits. Envelopes of forces created by these loads can be found using finite element elastic analysis of the system subjected to live and seismic actions using elastic response spectrum.

The example of analysis of composite steel-concrete spatial frame system with partial redistribution of moments subjected to seismic action is introduced.

АННОТАЦИЯ

Расчет зданий и сооружений на сейсмическое воздействие является одной из наиболее сложных задач в инженерной практике. Существует несколько методов расчета на сейсмическое воздействие, таких как расчет с применением упругого спектра реакций, нелинейный статический расчет, нелинейный метод прямого интегрирования во времени с применением акселерограмм произошедших землетрясений или искусственных акселерограмм и другие. Расчет зданий и сооружений на сейсмическое воздействие может быть выполнен в рамках решения оптимизационной задачи при анализе приспособляемости с учетом нелинейных характеристик материалов. Такой подход имеет ряд преимуществ. Внешние воздействия представляются как область нагружений, в результате решается задача для всех направлений сейсмического воздействия и для любой схемы полезной нагрузки одновременно. Как часть решения оптимизационной задачи учитывается упруго-пластическое и хрупкое поведение материалов.

В данной статье представлено новое решение для расчета сталежелезобетонных каркасных зданий с упруго пластическими и хрупкими элементами. Поведение железобетонных и сталежелезобетонных элементов под воздействием поперечного усилия предполагается хрупким. С другой стороны, поведение элементов под воздействием изгибающего момента предполагается упруго-пластическим.

Предполагается, что нагрузка изменяется случайно в заданных пределах. Огибающие усилий от этих нагрузок могут быть найдены при помощи упругого КЭ анализа системы на временные *и сейсмические воздействия с использованием упругого спектра реакций.*

Приведен пример расчета сталежелезобетонной пространственной каркасной системы с частичным перераспределением пластических усилий в результате сейсмического воздействия.

Keywords: composite steel-concrete frame, push-over analysis, seismic action, plastic hinge, response spectrum.

Ключевые слова: сталежелезобетонный каркас, нелинейный расчет, сейсмическое воздействие, пластический шарнир, упругий спектр реакций.

INTRODUCTION

The earthquake effect analysis for buildings and structures is one of the most difficult tasks in practical engineering. There are several methods to calculate earthquake effects, such as the method of direct integration of earthquake accelerograms, push-over analysis, and elastic response spectrum analysis. In this article, the shakedown analysis is applied for these purposes. Non-linear properties of materials can be taken into account when using any of these methods. For example, the non-linear (plastic) properties of materials are taken into account through the use of the behaviour coefficient q [8] when applying elastic analysis based on the response spectrum. In case of the use of the push-over analysis, the non-linearity of materials is taken into account directly by «inserting» plastic centroids. A more accurate solution, taking into account the non-linear strength and dynamic properties of materials can be obtained by applying the method of direct integration of earthquake accelerograms. Shear resistance of elements must be checked using any of these methods, after which some elements must be modified in order to increase the shear resistance with the subsequent re-analysis of the entire system. In the case of larger-sized systems, these checks may require several iterations, which can significantly increase the estimated time as a result.

The analysis for the earthquake effect of buildings and structures can be carried out in the furtherance of the optimization problem solution taking into account the non-linear properties of materials [2, 10]. This approach has several advantages:

- External influences are represented as a loading area, which allows solving the problem for all directions of earthquake impact and for any load application diagram simultaneously.
- In the process of solving the optimization problem, the elastoplastic and brittle behaviour of materials can be taken into account in the analysis [9].

The article presents a mathematical model for the analysis of buildings and structures containing elastoplastic and brittle elements. The load is assumed to vary randomly within the specified limits. These limits are determined by the direction and the magnitude of earthquake loads that can be found by linearly elastic analysis of the system using the elastic response spectrum.

An example of the shakedown analysis of a three-dimensional composite frame system with a partial redistribution of plastic forces as a result of earthquake effect is given.

MATHEMATICAL MODEL FOR THE ANALYSIS OF REINFORCED-CONCRETE SKELETONS

Let us assume that the problem of determining the carrying capacity of the structures under consideration is a general dynamic shakedown problem. First, we must solve equations of motion for a damped discrete elastic system under the load *F(t)* as a function of time *t*. This vector belongs to the loading region $\Omega(F(t), t)$, which is usually non convex but is approximated as convex in this case. The «elastic» solution is further used as a basis for calculating the inelastic system with allowance for partial plastic redistribution of forces. That is, the task of determining the load-carrying capacity of systems consisting of ideally plastic and brittle elements under the impact of variable loads is formulated as follows [2]. It is necessary to find the parameter (the safety factor) μ for the load vector F, as well as the vector of residual forces S_p^r such that

$$\mu \rightarrow \max \qquad (1)$$

$$S^e(t) = f(\mu F(t)) \tag{2}$$

$$Kq + K^d E_p d = 0 \tag{3}$$

$$\varphi_{pl}(S^e(t) + S_p^r, S_{0,pl}) \leq 0 \tag{5}$$

$$\varphi_{br}(S^e(t), S_{0,br})_i \leq 0, \ i \in I_{br} \tag{6}$$

$$F(t) \in \Omega(F_j(t), t) \tag{7}$$

where $S_{0,\ pl}$ and $S_{0,\ br}$ are vectors of limit internal forces in the cross sections of elastic-plastic and elastic-brittle elements accordingly;

S^e, S^r are vectors of elastic and residual forces in sections of elements;

$F_j(t)$ are vectors of j combinations of loads, $j \in J$;

J is the set of combinations of loads;

I_{br} is the set of i brittle elements;

$\Omega(\cdot)$ is the set of loads F;

t is the time;

K is the stiffness matrix;

q is the vector of unknown FEM (usually, the displacement vector),

K_d is the matrix of the impact of distortions d on the response in finite elements.

The indices *pl* and *br* refer to the elastoplastic and elastically brittle elements, respectively; the indices e and r – to elastic and residual forces.

E_p is the matrix that assigns the necessary values for residual internal forces (their presence or absence) in elastoplastic or elastically brittle elements, respectively (i.e., it determines a partial plastic redistribution of forces). The E_p matrix forming diagram is given below:

$$E_p = \mathrm{Diag}\begin{bmatrix} 1 \ \text{if plastic element} \\ 0 \ \text{if brittle element and } \varphi_{br}(\cdot)_i = 0, \ i \in I_{br} \end{bmatrix} \tag{8}$$

BEHAVIOUR OF ELEMENTS

For an elastoplastic element in bending, the diagram of the bending moment and the angle of rotation is shown in Fig. 1. For an elastically brittle element in the presence of sudden collapse, the diagram is shown in Fig. 2.

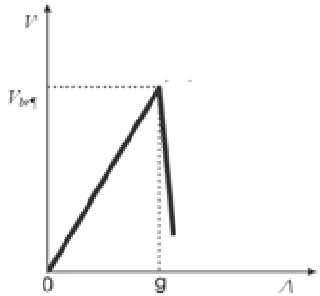


Figure 1. Cross-sectional force-displacement diagram for brittle elements

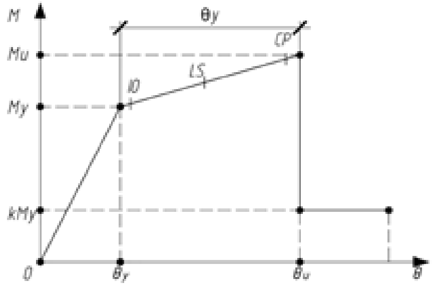


Figure 2. Moment-angle diagram for a plastic element

In the case of brittle fracture (for example, under the action of normal or transverse forces), inequality constraints (5) can be written down, respectively, as

$$N_{ed} \geq N_{rd}(M_{ed}) \text{ or } V_{ed} \geq V_{rd} \tag{9}$$

V_{ed} = the transverse force in the element section; V_{rd} = the shear strength of the element; N_{ed} = the normal force in the element section; $N_{rd}(M_{ed})$ = the tensile/compression strength of the element adjusted for the N-M interaction diagram.

No additional conditions other than (5, 7) are required when plastic centroids which condition is controlled by the force (N, V) emerge in the section [10].

Distortions *d* here are viewed as curvatures/angles of rotation in element sections. Due to this, it is also possible to limit the deformations in plastic centroids under the impact M. For this purpose, there is an additional inequality constraint:

$$\varphi_{pl}(\theta^e(t) + \theta_p{}^r(q), \theta_u) \leq 0\,, \tag{10}$$

where θ^e = the vector of elastic angles of element sections rotation; θ_p^r = the vector of plastic angles of element sections rotation.

EXAMPLE OF SHAKEDOWN ANALYSIS FOR A THREE-DIMENSIONAL COMPOSITE FRAME

The FEM Model

An example of the analysis of a three-dimensional composite frame with elastic-plastic and elastic-brittle elements is given below.

The dimensions of the frame are shown in Figure 3.

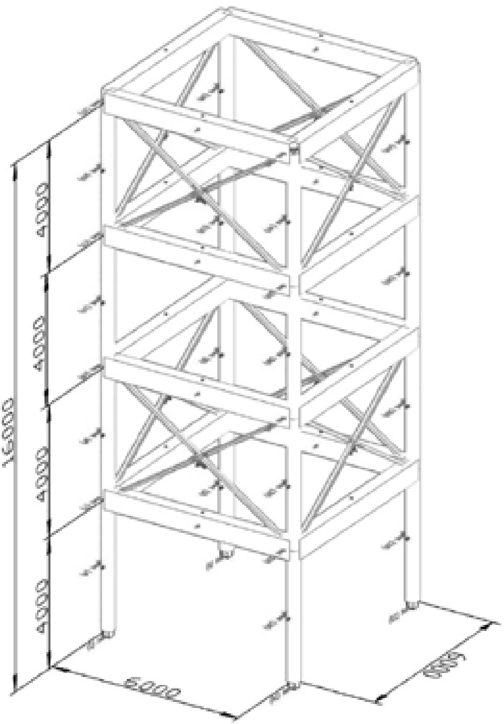


Figure 3. Composite three-dimensional frame

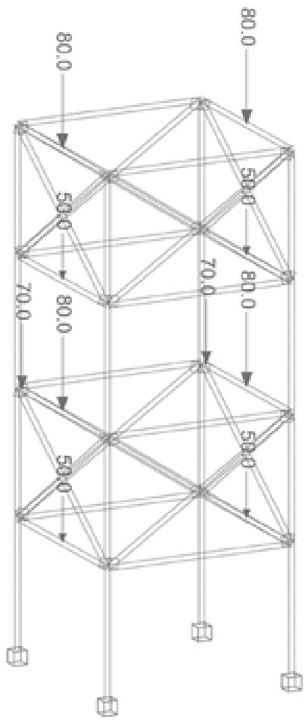


Figure 4. Variable load on the frame, kN

Earthquake effect and calculated combination of impacts

The first step is to determine the envelope of internal forces from earthquake effect in the elastic work stage. The earthquake effect is presented in the form of an elastic response spectrum. As an example, an elastic response spectrum for the D-type soil as per Eurocode 8 is adopted. The unit of soil acceleration is 1 m/s^2.

Forces and movements caused by earthquake impacts are determined as follows:

1. System properties are determined
   - The mass matrix m and the stiffness matrix k are determined
   - The damping coefficient ζ_n is determined (in this example, it is adopted at a rate of 5%)
2. The natural oscillation frequencies ω_n and the natural oscillation modes φ_n are determined
3. Peak responses for each waveform are calculated:
   - In accordance with the oscillation period T_n and the damping coefficient ζ_n, A_n (the ordinate of accelerations) and D_n (the ordinate of displacements) are determined using the elastic response spectrum
   - Displacements along the floors are determined using the formula $u_{jn} = \Gamma_n \varphi_{jn} D_n$
   - Equivalent static loads f_n are determined using the formula $f_{jn} = \Gamma_n m_j \varphi_{jn} A_n$
   - Response and internal forces in elements are determined using the elastic analysis for static loads f_n
4. Maximum effects from earthquake impact are determined by combining the maximum effects r_n for each waveform using the square root of the squared sum (SRSS) or the complete quadratic combination (CQC) rules.

For each critical loading, the calculated value of internal forces is found by combining the effects of impacts for an earthquake design situation in accordance with Eurocode 0 [5]:

$$Ed = G_{kj,\sup}\left(G_{kj,\inf}\right) + A_{ed} + \psi_{2,i} Q_{k,i},$$

where $G_{kj,\,sup}$ ($G_{kj,\,inf}$) is the unfavourable (favourable) characteristic constant effect (figure);

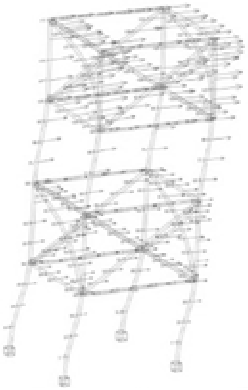
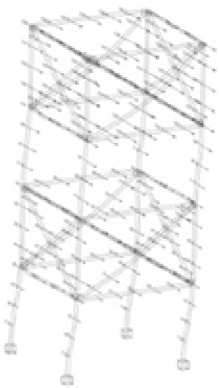
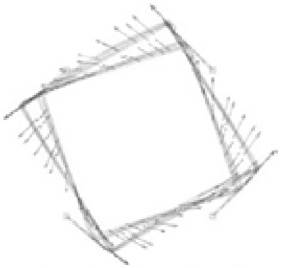
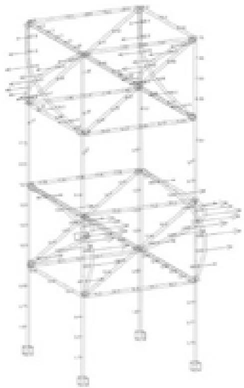
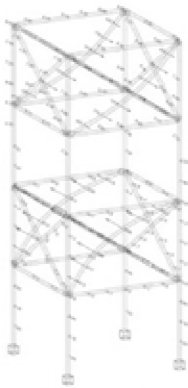
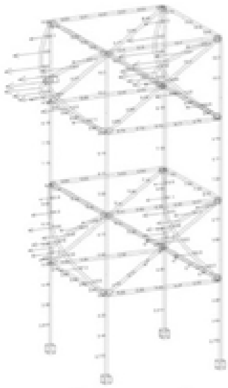
A_{ed} is the design earthquake effect;
$\psi_{2,\,i}$ is the coefficient in accordance with A1.2.2 of Eurocode 0 [5];
$Q_{k,\,i}$ is the corresponding variable impact (Figure 4).

Properties of materials and elements

The analysis model is shown in Figure 3. The cross sections of the frame elements are given in Table 2; the concrete strength class is adopted as C30/37.

Table 1

Own fluctuations

1  1.96 Hz	2  1.98 Hz	3  2.6 Hz
4  3.95 Hz	5  3.96 Hz	6  3.99 Hz

Calculated characteristics of elements (sections) for the non-linear analysis were determined according to the Global Resistance Factor method described in Fib Model Code 2010 [13] as follows [14]:

$$R_d = R\,(f_{cR,}\, f_{yR}, f_{uR})/\gamma_{R,} \tag{12}$$

where γ_R is the global safety factor equal to 1.3;

$$f_{cR} = 0.85 \cdot \alpha \cdot f_{ck};\ \alpha = 1;\ f_{yR} = 1.1 \cdot f_{yk},\ f_{uR} = 1.08 \cdot f_{yR}.$$

The stress-strain diagram for concrete is assumed to be parabolic in accordance with Eurocode 1992–1–1 but has been modified for use in non-linear ULS analyses. The stress-strain state diagram for reinforcing steel is considered bilinear, modified for non-linear computations in ULS. Modified diagrams of deformation for concrete and reinforcement are shown in Fig. 5, 6.

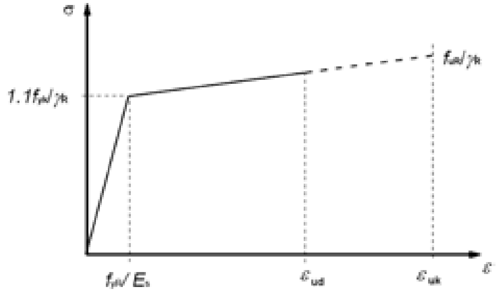


Figure 5. Deformation diagram for reinforcing steel

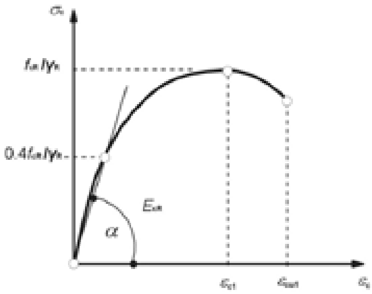
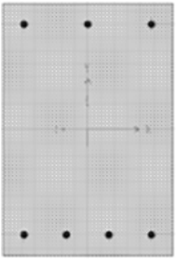
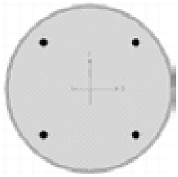
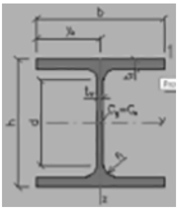


Figure 6. Deformation diagram for concrete

For a high intensity earthquake, if the rate of increase in compressive stresses or deformations is in a constant range of approximately 1 MPa/s $< |\sigma_c| < 10^7$ MPa/s and $30 \cdot 10^{-6}\ s^{-1} < |\varepsilon_c| < 30 \cdot 10^{-2}\ s^{-1}$, the provisions given in sub-clause 5.1.11.2.1 of the Fib Model Code 2010 can be used. With their help, we can account the effects of stress or strain rates.

Table 2

Sections of frame elements

No.	Section	b(D), mm	H, mm	Longitudinal reinforcement, class		Profile, class
				upper	lower	
1	 Beam	400	600	3Ø16, S500	4Ø16, B500B	
2	 Column	400		4Ø16, S500		Pipe 377x6, C235
3	 Links					I-beam 100, C255

The plastic bearing capacity for bending of all the composite concrete elements was calculated on the basis of the analysis of the «earthquake moment-curvature (rotation angle)» constraint according to [7]. The earthquake moment / rotation angle curvature can be idealized in the form of an elastically ideally plastic constraint for estimating the plastic bearing capacity of the section of the element [3]. The plastic bearing capacity of sectional elements for bending is shown in Table 3.

For columns, it is necessary to take into account the possibility of both brittle fracture due to concrete crushing during compression and plastic fracture due to the plastic flow of stretched reinforcement steel. This requires constructing an N-M constraint diagram for the elements under compression and bending. The N-M constraint is taken into account both in analysing the earthquake moment (curvature) for the plastic bearing capacity and in solving the optimization problem while taking into account the brittle fracture condition (8).

Table 3

Plastic bearing capacity of element sections in terms of the earthquake moment

Cross-section number in Table 1 (longitudinal force N, *kN*)	**Plastic bearing capacity M_p for the positive earthquake moment, *kN•m***	**Plastic bearing capacity M_p for the negative earthquake moment, *kN•m***
1	192	146.5
2 (–574)	284	284
2 (–197)	276	276

Transverse reinforcement of all reinforced-concrete beams is made of Ø8 S500 reinforcement steel in 200 mm increments. Structural drawings are shown in Figure 9. The shear resistance of reinforced-concrete beams is calculated according to Eurocode 2 [6]. The bearing capacity for shearing $V_{Rd,\,s}$ was 226.7 kN.

The shear resistance of composite columns is calculated according to Eurocode 4 [7]. The separation of the transverse force V_{Ed} into the components $V_{a,\,Ed}$ and $V_{c,\,Ed}$ acting on the steel section and the reinforced-concrete core, respectively, is taken in the same proportion as the distribution of the bearing capacity over the bending moment $M_{pl,\,Rd}$ between the steel section and the reinforced-concrete core (see Table 4). Structural drawings of the frame assembly are shown in Figures 7 and 8.

Table 4

Shear resistance of reinforced-concrete columns

Part of section	**Shear resistance, V_{Rd}, *kN***	**Resistance to the bending moment of flection $M_{pl,\,Rd}$, $kN \bullet m$**	**Transverse force (max) V_{Ed}, *kN***
Concrete core	296	102	30.2
Steel pipe	630	202	60.4

The earthquake moment envelop curve in the elastic work stage is shown in Figure 11.

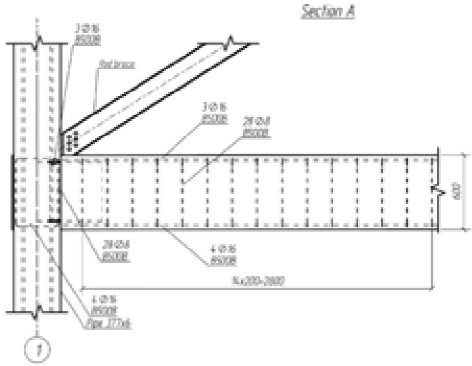


Figure 7. Drawings of the frame assembly structures

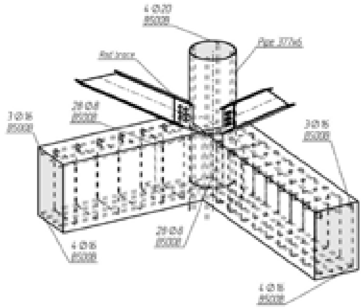


Figure 8. View of the frame assembly

Results of shakedown analysis of the three-dimensional frame

The envelop curve of shear forces is shown in Figure 9.

The envelop curve of longitudinal forces is shown in Figure 10.

The envelop curve of moments in the elastic work stage is shown in Figures 11 and 12.

To solve the optimization problem, first a vector of residual forces from unit deformations (curvature) k_r in the cross sections of elastoplastic elements must be found.

The non-linear optimization problem was solved using the «Mathematica» package in the form of several sequential linear programming problems. The impact of the longitudinal force (Figure 10) on the carrying capacity in terms of the earthquake moment was taken into account in the second iteration, which resulted in the load reserve factor of $\mu = 1.12$.

The number of unknown values under the Finite Element Method for solving the optimization problem was 1,500. The matrix of constraints/inequalities for this frame has dimensions of 448x57; the solution of each linear optimization problem takes 0.016 s.

Shakedown analysis results are shown in Figures 13 and 14. Comparison with the elastic analysis (without the safety factor) is given in Table 5. The shakedown analysis demonstrates the redistribution of earthquake moments from the outermost sections of the lower beam to the central ones, which optimizes the use of the beam bearing capacity.

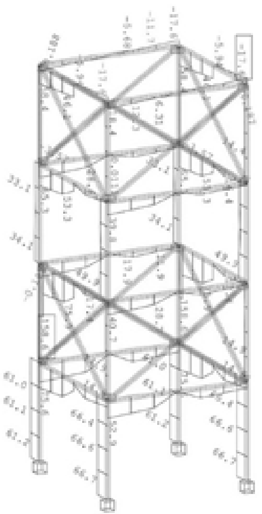


Figure 9. Envelop diagram (max. value) of shear forces, *kN.*

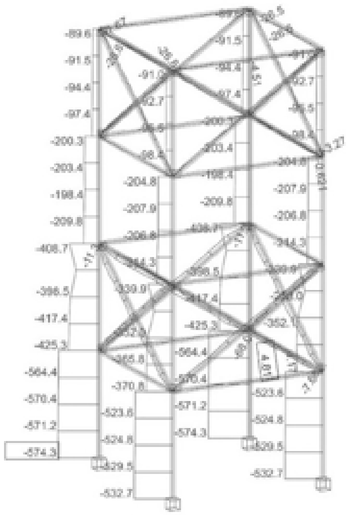


Figure 10. Envelop diagram (min. value) of longitudinal forces, *kN.*

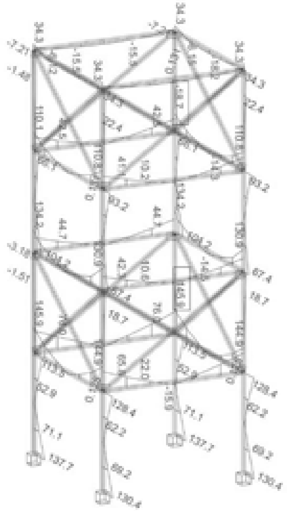


Figure 11. Envelop diagram (max. value) of earthquake moments, *kNm.*

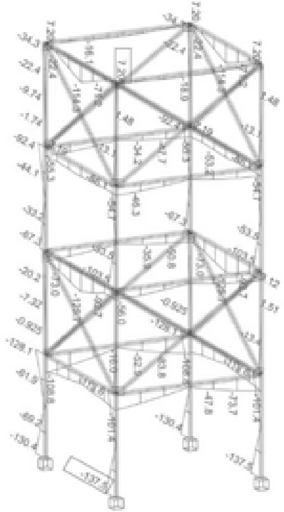


Figure 12. Envelop diagram (min. value) of earthquake moments, *kNm.*

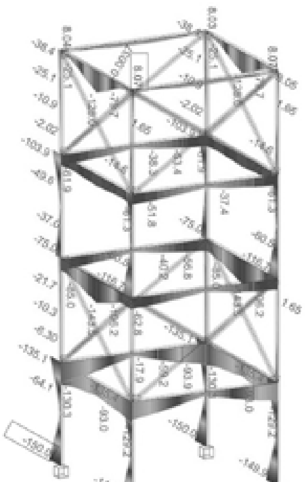


Figure 13. Envelop diagram (min. value) of earthquake moments after plastic redistribution, kNm.

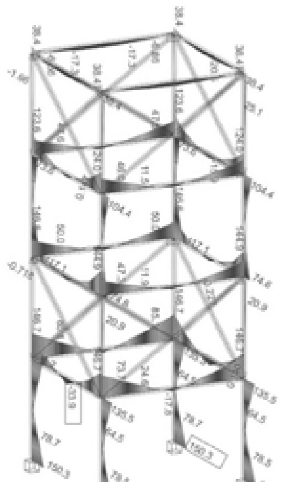


Figure 14. Envelop diagram (max. value) of earthquake moments after plastic redistribution, kNm.

Table 5

Comparison of results

Result	Elastic analysis	Analysis of shakedown (safety factor 1.12)
Earthquake moments in the sealing of the bottom column Mmax, kN•m	137.7	150.3
Earthquake moments in the sealing of the bottom column Mmin, kN•m	–130.4	–150
Earthquake moments in the centre of the lower beam Mmin, kN•m	–54.2	–93
Earthquake moments in the right part of the lower beam Mmin, kN•m	–113.6	–129.2
Earthquake moments in the right part of the lower beam Mmax, kN•m	145.9	146.7

SHAKEDOWN ANALYSIS SEQUENCE

Summarizing the above results allows representing the general sequence of shakedown analysis for earthquake effects. A block diagram of this analysis is given in Figure 15.

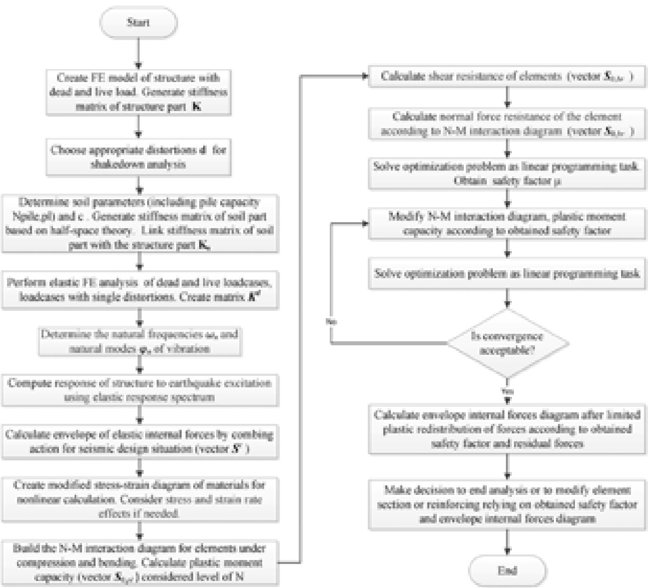


Fig. 15. General block diagram of shakedown analysis for earthquake effects

CONCLUSION

This article proposes a modification of the authors' previous mathematical model [2] for the optimization problem of calculating systems with elastoplastic and elastic-brittle elements. This model is modified for use in conjunction with a complex of FE Analysis without additional application of the methods of construction mechanics. The application of improvements has allowed introducing an additional restriction (10) for section rotation angles for plastic centroid condition controlled in terms of deformation. In addition, integration with the Finite Element Method has significantly increased the dimensionality of the solved problems, including the solution of problems in a three-dimensional setting without significantly increasing the required labour intensity. This is supported by an example of shakedown analysis for a three-dimensional composite frame under earthquake effects and an additional safety factor of the bearing capacity of the system.

This model is designed for an optimal design of composite and reinforced-concrete frame structures under earthquake effects. It allows revealing additional reserves of the bearing capacity, which is important for an analysis in special design situations (including those at the impact of earthquake). Further development of the model will allow solving the inverse problem of optimal selection of elements and fittings sections (as well as tasks of the topological optimization of the structure).

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