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MODAL ANALYSIS OF THREE-DIMENSIONAL FINITE ELEMENT MODELS OF REINFORCED CONCRETE STRUCTURES

ABSTRACT

In this article modal analysis of reinforced concrete structures by the Finite Element Method is reviewed. A list of simplifications – implemented within idealization of the structural scheme at the calculation phase – that reduce the reliability of the results is provided. Out of the necessity to use idealizations, the Finite Element Method allows for the possibility of creating multiple alternative computational models for each structural model respectively, and there are many alternative calculation results. Consequently, the calculation of reinforced concrete structures always leads to more than a single result. Idealization of geometric shape has the greatest influence on the variability of results. Accordingly, the need to use solid finite elements in calculations of reinforced concrete structures including rod and slab elements becomes apparent. This will provide means to abandon the subjective construction of the design model of the structure. In addition to traditional calculations of the stress-strain state from the action of static load for a number of structures, a modal analysis should be performed (dynamic calculation). Its purpose is to determine the shapes and free oscillation frequencies in order to determine the correctness of the structural scheme by the first shapes and compare the values of natural frequencies with the regulatory requirements. Since the significant need for computational resources poses a natural limitation of the use of solid finite element models for modal analysis, the effectiveness of this method of calculation should be established. The article showed the effect of applying the idealization of the geometric shape on the *results of the modal analysis by comparing two calculations of the FEM of one structure and concluded that solid finite elements should be used for the calculation of reinforced concrete frame structures.*

Keywords: reinforced concrete, the Finite Element Method, solid finite elements, modal analysis

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МОДАЛЬНЫЙ АНАЛИЗ ПРОСТРАНСТВЕННЫХ КОНЕЧНО-ЭЛЕМЕНТНЫХ МОДЕЛЕЙ ЖЕЛЕЗОБЕТОННЫХ КОНСТРУКЦИЙ

АННОТАЦИЯ

В статье рассматривается проблема выполнения модального анализа железобетонных конструкций методом конечных элементов. Приводится перечень упрощений, реализуемых в рамках идеализации конструктивной схемы на стадии выполнения расчетов, которые снижают достоверность результатов. Метод конечных элементов по причине необходимости использования идеализаций допускает возможность создания множества вариантов расчетных моделей для одной и той же конструктивной модели, соответственно, существует и множество близких вариантов результатов расчета. Таким образом, расчет железобетонных конструкций всегда допускает не единственный результат. Наибольшее влияние на *вариабельность результатов оказывает идеализация геометрической формы. Соответственно, очевидным становится использование объемных конечных элементов при расчетах железобетонных конструкций, состоящих в том числе из стержневых и плитных элементов. Это позволит отказаться от субъективного построения расчетной модели конструкции. Помимо традиционных расчетов напряженно-деформированного состояния от действия статической нагрузки для ряда конструкций необходимо выполнение модального анализа (динамического расчета). Его цель состоит в определении форм и частот собственных колебаний с тем, чтобы по первым формам определить корректности конструктивной схемы и сравнить значения собственных частот с нормативными требованиями. Поскольку естественным ограничением использования объемных конечно-элементных моделей для модального анализа является значительная потребность в вычислительных ресурсах, необходимо установить эффективность данного метода расчета. В статье на сопоставлении двух расчетов МКЭ одного и того же сооружения показано влияние применения идеализации геометрической формы на результаты модального анализа и сделан вывод о необходимости использования объемных конечных элементов для расчета каркасных железобетонных конструкций.*

Ключевые слова: железобетон, метод конечных элементов, объемные конечные элементы, модальный анализ

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INTRODUCTION. THE PROBLEM OF IDEALIZATION OF REINFORCED CONCRETE STRUCTURE MODELS

Reducing errors when performing calculations of reinforced concrete structures and increasing the correctness and reliability of the final results requires the use of a minimum number of simplifying assumptions and premises. The main difficulty lies in the preparation of the design model, implementation and evaluation of calculations of reinforced concrete structures using the Finite Element Method (FEM). A key factor in modeling a real structure is the need for idealization:

1) of a geometric shape, which provides conditional division of a model into three-dimensional (soil foundation), two-dimensional (floors and walls) and linear (columns and beams) elements;

2) of a structure, where it is impossible to take into account absolutely all the properties of the structures in the model, so it is simplified when taking into account the basic physical properties;

3) of kinematic relations and external force actions, where the nodes of the connecting elements and the boundary conditions in most cases are difficult to describe numerically.

Real elements of buildings or structures have shapes and sizes, while at the same time, in order to simplify modeling and reduce the consumption of computing resources, in general, finite elements (FE) with infinitely small thicknesses are used. Dimensions of a reinforced concrete structure together with characteristics of concrete are considered as indicators of stiffness of the section. For example, any column is modeled by a finite element in the form of a line with a cross section in the form of an infinitely small point, and the floor panel is in the form of a slab with infinitely small thickness. This approach allows us to significantly simplify mathematical models and perform selection of reinforcement, while their dimensions are minimal. The disadvantage of this approach is the need to use finite elements that describe physical essence of behavior of the loaded structure as closely as possible, which can pose difficulties in assessing the impact of dimensions of elements on this ratio. Idealization greatly simplifies the calculations, but leads to a decrease in their accuracy and the need for additional evaluation and interpretation. For example, a surge in the values of force appears due to the idealization of structural elements by assigning to them infinitely small cross-sectional sizes in the calculation model. No such phenomena is observed in the real structures. Various methods of smoothing such effects require substantial efforts and provide a subjective implementation [1], and also require appropriate qualifications of the analysts. The main disadvantage of this approach is the adoption of assumptions involving the use of so-called “perfectly rigid bodies”. As a result of applying this approach to the calculations of reinforced concrete structures, in the design process – from the point of developing a structural model of an object to obtaining the final working documentation – a series of “transitions” is required between conditionally three-dimensional and two-dimensional (or linear) representations of the object and its elements, which requires the participation of qualified engineers to perform calculations, and leads to an infinite variation of the calculation results.

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The apparent and simplest solution to this problem is the use of solid finite elements for the calculation of reinforced concrete frame buildings. To date, bulk finite elements are applicable even to the modeling of thin three-dimensional shell-like structures of different curvature [2]. However, for the purpose of mass design, modeling by solid finite elements is associated with a number of computational difficulties associated primarily with the size of the resolving matrix of equations, and the need to comply with the design standards when selecting reinforcement.

The above will be confirmed by analyzing the results of dynamic calculations of a single-story building with a reinforced concrete frame according to two models. Note that no such comparison is known to have been made by any author in the scientific literature.

- traditional model – created from FE rods and shells of zero curvature (Figure 1);
- model created from parallelepiped FE (Figure 2).

External load is represented by the own weight of the building.

As is known, free oscillations of a linear system with n degrees of freedom without taking into account resisting forces are described by a system of ordinary differential equations.

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\},$$

where u is a vector–column of displacement;

$[M]$ and $[K]$ are mass and stiffness matrices, respectively.

As a result, with harmonic oscillations of a building, this task is reduced to the search for eigenfunctions and eigenvalues of the determinant of a system of resolving linear algebraic homogeneous equations [3].

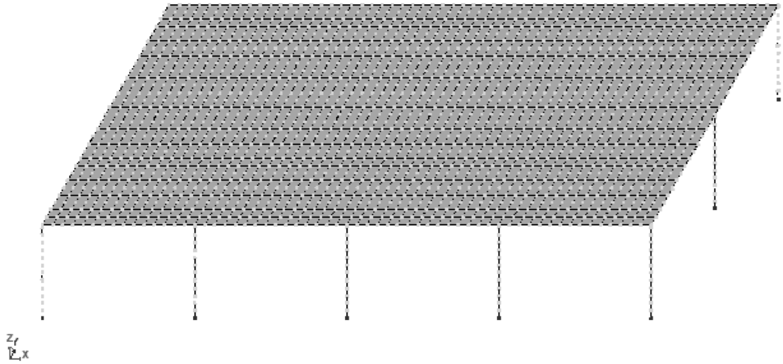

Figure 1. Traditional finite element model of a one-story building

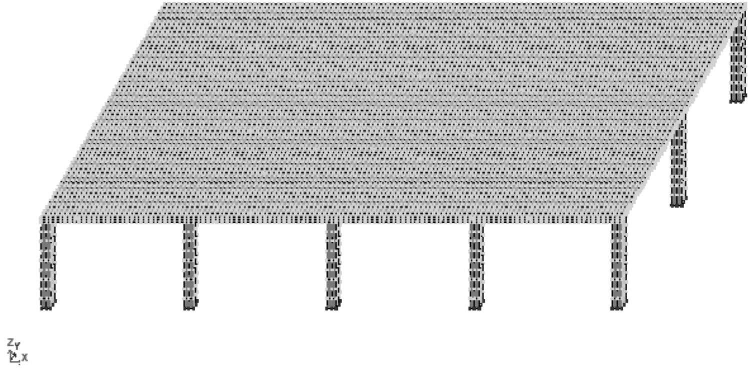

Figure 2. Finite element model of a one-story building of solid FEs

It should be noted that attempts to bring the masses to the nodes of the FE model will prevent the correct modal analysis of the system. The only way to reduce the amount of computation is to reduce the number of nodes in the system, provided that there is no loss of accuracy. Comparison of the results of calculations of these two finite element models confirm the above statement (Tables 1 and 2).

Table 1

Eigenvalues, eigenfrequencies, periods of oscillations of the finite element model shown in Figure 1

Item No.	Eigenvalues	Frequencies		Periods	Distribution Coefficient	Modal Mass	
		rad/s	Hz	s		in %	
1	0.039263	25.47	4.06	0.2466	4.034301	32.2	32.2
2	0.039217	25.50	4.06	0.2463	3.938469	30.7	62.9
3	0.033024	30.28	4.82	0.2074	-0.004602	0.0	62.9
4	0.017197	58.15	9.26	0.1080	0.125175	0.0	62.9
5	0.016966	58.94	9.39	0.1065	-0.046691	0.0	62.9
6	0.016855	59.33	9.45	0.1058	-0.097134	0.0	62.9
7	0.016721	59.80	9.52	0.1050	-1.973780	7.7	70.6
8	0.015525	64.41	10.26	0.0975	0.036006	0.0	70.6
9	0.015266	65.51	10.43	0.0959	0.009510	0.0	70.6
10	0.014696	68.05	10.84	0.0923	-3.052237	18.4	89.1
11	0.014192	70.46	11.22	0.0891	-0.037909	0.0	89.1

Table 2

Eigenvalues, eigenfrequencies, periods of oscillations of the finite element model shown in Figure 2

Item No.	Eigenvalues	Frequencies		Periods	Distribution Coefficient	Modal Mass	
		rad/s	Hz	s		in %	
1	0.029806	33.55	5.34	0.1872	4.086401	31.8	31.8
2	0.029671	33.70	5.37	0.1863	-4.059780	31.4	63.2
3	0.025190	39.70	6.32	0.1582	0.014038	0.0	63.2
4	0.013203	75.74	12.06	0.0829	0.252577	0.1	63.2
5	0.013034	76.72	12.22	0.0819	0.064970	0.0	63.3
6	0.012976	77.07	12.27	0.0815	-0.119207	0.0	63.3
7	0.012872	77.69	12.37	0.0808	-1.851280	6.5	63.3
8	0.012312	81.22	12.93	0.0773	0.001081	0.0	69.8
9	0.012124	82.48	13.13	0.0761	-0.097063	0.0	69.8
10	0.011730	85.25	13.57	0.0737	-3.162998	19.0	88.9
11	0.011491	87.02	13.86	0.0722	0.048630	0.0	88.9

CONCLUSION

Comparison and analysis of the results of calculations in Tables 1 and 2 demonstrates the following:

1) the types of eigenforms match, but in the first two flexural modes of oscillations, the directions of oscillations differ;

2) the distribution of local maxima of modal masses match with the numbers of eigenfrequencies;

3) there are significant differences in the values of eigenfrequencies – up to 32 %;

4) the idealized model shows a partial loss of the vertical load from the dead weight;

5) the type and nature of the distribution of static deformations in the models are the same, while the absolute value of the maximum deflections differ by up to 67 %.

Obviously, the difference in computational models can seriously affect the values of the determined dynamic forces and displacements in dynamic calculations. Therefore, a significant influence of the idealization of the geometric shape of the reinforced concrete structure on the results of the modal analysis has been established. In practice, this circumstance should be taken into account when calculating objects for which the frequency value of the first mode of oscillation is normalized.

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