DOI: 10.23746/2018-10-04

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ABOUT IMPLEMENTATION NECESSITY INTO DESIGNS OF EXTREME STRENGTH CRITERION INSTEAD OF THE DEFORMATION ONE

ABSTRACT

It is grounded the general strength design method on the cross (normal) sections which allows to solve any practical problems for the bending and eccentrically compressed-tensile RC elements up to the uniaxially compressed ones. Such method ought to use the complete set of Continuum Mechanics Equations (dynamic-static, geometric, constitutive for concrete and steel) and additional certain Strength Criterion (SC). In the current Codes all over the world the SC is applied as the known Deformation Strength Criterion (DSC). The DSC historic sources are analyzed and it is shown that its basic statement to find the concrete ultimate strain ε_{cu} *for cross sections of RC elements from the descending branch of concrete compression diagram is wrong in consequence the series of causes and in particular because the* ε_{cu} *value is defined not only concrete properties but other cross section conditions too: reinforcement quantity and its tension diagram type, section shape, load character etc. Therefore it is grounded the new SC from consideration the development of stress-strain state in the uneven compressed concrete zone of RC element during loading with taking into account of concrete peculiarities as so called «pseudo(quasi)plastic» material. The new «Extreme Strength Criterion (ESC)» together with Continuum Mechanics Equations leads to the General Strength Design Method of Normal Sections (GSDMNS), which overcomes the above demerits of DSC and designs based on the* *one. The GSDMNS merits, design algorithms, software and proximity of theoretic and experimental values are discussed.*

Keywords: RC element, normal section, strength criterion, concrete compression zone, ultimate strain, pseudo(quasi)plasticity peculiarities, maximum load criterion, optimization design.

For citation: Mitrofanov V., Pinchuk N. About implementation necessity into designs of extreme strength criterion instead of the deformation one. *Contemporary Issues of Concrete and Reinforced Concrete: Collected Research Papers*. Minsk. Institute BelNIIS. Vol. 10. 2018. Pp. 58–77. https://doi.org/10.23746/2018-10-04

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О НЕОБХОДИМОСТИ ВНЕДРЕНИЯ В РАСЧЕТЫ ЭКСТРЕМАЛЬНОГО КРИТЕРИЯ ПРОЧНОСТИ ВЗАМЕН ДЕФОРМАЦИОННОГО

АННОТАЦИЯ

Обосновывается общий метод расчета прочности по нормальным сечениям, который позволяет решать разнообразные практические задачи для изгибаемых и внецентренно сжатых-растянутых железобетонных элементов вплоть до центрально сжатых. Такой метод должен использовать полную систему уравнений механики сплошной среды (динамические – статические, геометрические, физические для бетона и арматуры) и некоторый дополнительный критерий прочности (КП). В действующих нормах во всем мире применяется в качестве КП известный Деформационный Критерий Прочности (ДКП). Анализируются исторические источники ДКП. Показывается, что его основное положение по определению предельной деформации бетона εcu для нормальных сечений железобетонных элементов по нисходящей ветви диаграммы сжатия бетона является ошибочным вследствие ряда причин и, в частности, потому, что величина ε_{cu} определяется не только свойствами бетона, но и другими условиями поперечного сечения: количеством арматуры и типом ее диаграммы растяжения, формой сечения, характером нагрузки и др. Поэтому обосновывается новый КП из рассмотрения развития напряженно-деформированного состояния в неравномерно сжатой зоне бетона железобетонных элементов в процессе нагружения с учетом особенностей бетона – так называемого «псевдо(квази)пластического» материала. Новый «Экстремальный Критерий Прочности (ЭКП)» вместе с уравнениями механики сплошной среды приводит к Общему Методу Расчета Прочности Нормальных Сечений (ОМРПНС), который преодолевает недостатки ДКП и расчетов, основанных на нем. Обсуждаются достоинства ОМРПНС, алгоритмы расчета, программное обеспечение и близость теоретических и экспериментальных величин.

Ключевые слова: железобетонный элемент, нормальное сечение, критерий прочности, сжатая зона бетона, предельная деформация, псевдо(квази)пластичность, критерий максимума нагрузки, оптимизационный расчет.

Для цитирования: Митрофанов, В. П. О необходимости внедрения в расчеты экстремального критерия прочности взамен деформационного / В. П. Митрофанов, Н. М. Пинчук // Проблемы современного бетона и железобетона : сб. науч. тр. / Ин-т БелНИИС; редкол.: О. Н. Лешкевич [и др.]. – Минск, 2018. – Вып. 10. – С. 58–77. https://doi.org/10.23746/2018-10-04

INTRODUCTION

At present the strength design on normal sections of RC elements recommended by all countries Codes, including [1, 2], are based on the DSC, according to which the section failure arises when in the concrete compressed extreme fibers strain ε_{cm} reaches the so called «ultimate strain ε_{cu}» which is given in Codes depending on the concrete strength only. The ε_{cu} values may exceed considerably the strain ε_{c1} of uniaxially compressed concrete under the peak stress f_c that is conformed with test data obtained for the real RC beams and eccentrically compressed columns by the no small enough eccentricity e_0. Apparently, when $e_0 \rightarrow 0$, the $\varepsilon_{cu} \rightarrow \varepsilon_{c1}$ must be, but to this transition the DSC and designs based on the one do not lead. The last fact means the e_0 value (i.e. character of section stress-strain state) influences on the ε_{cu} value. Furthermore the experiments show the ε_{cu} value depends on significantly also longitudinal reinforcement quantity, its tension diagram character and shape of normal section [3].

The adduced facts testify that ultimate strain ε_{cu} is characteristic of cross section conditions as a whole, including its concrete, but not concrete properties only. Nevertheless to-day many specialists, including the MC 2010 [2] developers, deem the concrete ultimate strain ε_{cu} of cross sections in RC elements is determined only with the concrete peculiarities and ignore the experimental data showing the ε_{cu} value dependence on complex of the section conditions. Such situation has historic roots having been begun from surmise (Zaliger, 1932) and experimental evidence (Berg et al., 1966, 1969) of the concrete stresses decrease $\sigma_c < f_c$ near the concrete extreme fibers in RC beams before their failure. This fact showed the existence of the overultimate «descending branch» of concrete compression diagram and explained the necessity ultimate strains $\varepsilon_{cu} > \varepsilon_{\chi 1}$. On the other hand the above fact pushed to experimental research of the descending branch for what the various test installations were offered [4, 5]. The most prevailing ones were installations in which the concrete prism or cylinder was loaded parallelly with rigid enough steel elastic element. These installations were unable to secure the constancy of compressed specimens strains velocity during test. The hand-operated presses had the same demerit. Therefore without strains velocity control the deformation process was arbitrarily accelerated in some state on the descending branch and the sudden failure of being tested specimen took place. This failure was interpreted as «ultimate state on the descending branch» and the corresponding stress σ_{cu} and strain ε_{cu} began to consider as «concrete ultimate stress and strain on the descending branch» respectively. Herewith the above strain ε_{cu} was adopted as the DSC.

In that past times it was attached no importance to the strains velocity constancy as the obligatory demand to constancy of experiment conditions during test. But later the noted demerit was realized and perfect test installations were worked out as «Digital Closed-Loop Servo-Controlled Hydraulic Testing Machines» which effect the strict control of the specimen strains velocity. As a result the real complete concrete compression diagrams were obtained with descending branch without sudden concrete failure but with gradual asymptotic approach to the strains axis [6–8]. These experiments negate the mentioned above values ε_{cu}, σ_{cu} as ultimate characteristics of complete concrete compression diagram. Apparently together with negation of the above values ε_{cu}, σ_{cu} the DSC as a whole loses its validity. Thus it is necessary to substitute the unsound DSC for some another more adequate SC.

In the present paper the necessity of new «Extreme strength Criterion (ESC)» is grounded on the base of analysis the concrete pseudoplasticity display in the concrete non-uniformly compressed zone of RC elements. This paper is development of the work [9] where the ESC was put forward and the General Strength Design Method of Normal Sections (GSDMNS) is worked out. Therefore here the GSDMNS is stated the general outline whereas it is given main attention to making more clear the physical reality of concrete pseudoplastic features and RC elements failure process in order to designers and students can master the ESC and GSDMNS more profoundly and swiftly.

TYPES OF CONCRETE FAILURE AND FAILURE CRACKS

The concrete failure types (states) can be highly different depending on first of all the average stress (hydrostatic pressure) σ [10]:

1) brittle failure, when $\sigma_{3t} > \sigma > \sigma_{bbp}$; σ_{3t} – concrete strength under uniform three-axial tension, $\sigma_{bbp} \approx 0$ – boundary stress σ between brittle and pseudoplastic failure;

2) pseudo(quasi)plastic failure, when $\sigma_{bbp} \geq \sigma \geq -\sigma_m$, $\sigma_m < 0$ – boundary stress σ between pseudo(quasi)plastic and plastic failure;

3) plastic failure, when $\sigma < -\sigma_m \approx 20f_c$.

According to contemporary notions the brittle failure is connected with development of rupture crack which firstly grows up from the initial structure defect (microcrack usually) to the critical length and then one is suddenly converted into rupture macrocrack, dividing element on the parts. The brittle strength is determined in accordance with Fracture Mechanics [11].

The pseudoplastic failure takes place under compressive stresses σ of the middle values which are just characteristic of RC structures. Therefore the pseudoplastic failure counts for much but the one is complicated enough phenomenon which is accompanied by the volume increase (dilatation) in consequence of microckacking, display of descending branch of stress σ_{ij} – strain ε_{ij} relations, particular stresses redistribution, ESC etc. The Pseudoplasticity Theory stays insufficiently worked out part of Mechanics.

Under high enough pressure σ all materials show plasticity. This effect is observed also in structurally-heterogeneous materials (rock, concrete, cast iron etc.). In this case the high pressure stops the microcracking and dilatation and plastic strains become possible owing to the microshears between particles of material. When $\sigma < -\sigma_m$ the Theory of Plasticity [12] is applicable for the concrete plastic strength determination.

The cracks nearly always accompany all failure types especially in concrete and rock. Therefore profound enough notions about concrete failure can't dispense with crack types information. In general the crack is surface on which displacements have leap. Depending on direction of displacements leap relatively normal and tangent to crack surface axes the cracks are:

1) cracks of normal rupture have displacements leap only in normal to crack surface direction; such cracks break the continuity of material and are distinctive for brittle failure; the adjacent surfaces of such cracks do not interact;

2) cracks of shear have displacements leap only in tangent to crack surface direction; such cracks are distinctive for the plastic failure; these cracks do not break the continuity of material and keep the interaction of their adjacent surfaces;

3) cracks of mixed (shear-rupture) type have displacements leaps in normal w_{crc} and tangential Δ_{sh} to crack surface directions; these cracks are characteristic of the pseudoplastic failure; the ones lead as rule to breaking of the material continuity; the interaction (interlock) of the adjacent surfaces of mixed cracks may be both absent and present completely or partly along their length. The interlock in mixed cracks is defined on the one hand by the so called «Character of Crack Surface Roughness (CCSR)» and on the other hand by the relative crack faces movement, i.e. by the normal w_{crc} and tangential Δ_{sh} displacements of adjacent surfaces of mixed crack [13]. Herewith there is so called

«Criterion of Existence of Interlock in Crack (CEIC)» [14], which permits to reveal the interlock presence-absence in mixed cracks. It is necessary to emphasize the important feature of all crack types (the mixed and shear cracks especially): the ones are narrow enough layers in which the irreversible deformations are localized [15, 12].

OVERULTIMATE STATE OF COMPRESSED CONCRETE SPECIMENS

In failure state the concrete uniaxially compressed specimens are divided mainly by the mixed cracks with faces interlock conservation. In experiments it is observed two main structure cases of being failed specimens: 1) asymmetrical, 2) symmetrical (Fig. 1, a).

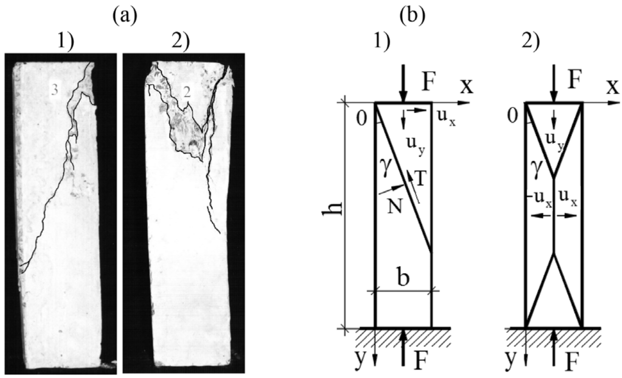

Figure 1. (a) Failed concrete prisms photographs of asymmetrical (1) and symmetrical (2) ultimate structures; (b) corresponding kinematic mechanisms.

These specimens structures can be considered as ultimate kinematic mechanisms (Fig. 1, b) in which separated by cracks rigid enough parts are mutually moved thanks to the irreversible strains localized in the cracks. Herewith the specimens parts have displacements u_x, u_y along corresponding axes x, y.

The needed test regime of concrete compressed specimens under overultimate state can be revealed from consideration of more simple case 1 of kinematic mechanism (Fig. 1, b, 1) when the movement equations of the top part are the following

$$M_1\ddot{u}_y = F(t) - \sum F_{int,y}\ ; \tag{1}$$

$$M_1\ddot{u}_x = \sum F_{int,x}\ , \tag{2}$$

where M_1 – mass of the specimen top part, $F(t)$ – external active force – load, $F_{int,y}$, $F_{int,x}$ – projections on the axes y and x respectively of tangential T and normal N forces of concrete internal resistance acting in the failure crack. Apparently the forces T and N in the concrete overultimate states are decreased as the displacements u_x, u_y and strains $\varepsilon_y = u_y/h$, $\varepsilon_x = u_x/b$ are increased. Then load F(t) must also decrease respectively in order to eliminate the specimen sudden failure. Herewith the equations (1), (2) show that static deformation of specimens will be reached if their right parts are equal to zero and displacements and stains velocities are constant during test

$$\dot{u}_y = \text{const}, \quad \dot{u}_x = \text{const}, \quad \dot{\varepsilon}_y = \dot{u}_y/h = \text{const},$$

$$\dot{\varepsilon}_x = \dot{u}_x/b\,\text{const}. \qquad (3)$$

During protracted enough period the used test installations did not observe the conditions (3).

For the conditions (3) observation it is necessary to set due value of strains velocity. The apt $\dot{\varepsilon}_y$ value can be estimated from consideration the stable concrete deformation on the ascending branch of its compression diagram under uniformly increasing load F(t)=at, where a – velocity of load increase. For the diagram peak point $F_m(t_m) = at_m = f_cA_c$, where t_m – specimen loading time up to peak stress f_c, A_c – specimen cross section area. As a result the average stain velocity is obtained

$$\dot{\varepsilon}_c = \varepsilon_{c1}/t_m, \qquad (4)$$

where $t_m \geq 20$ minutes is recommended [16].

It is obtained $\dot{\varepsilon}_c = (1,5...2,5)10^{-6}$ 1/s from (4). The experimentally found $\dot{\varepsilon}_c = (4...6)10^{-6}$ 1/s [6,7], i.e. the formula (4) gives the $\dot{\varepsilon}_c$ value of the same order with test one but the (4) is more prudent.

PSEUDO(QUASI)PLASTICITY

The mechanics of Pseudo(quasi)plastic materials is still being worked out. Therefore it is important to emphasize the peculiarities of these materials, which first of all are structurally-heterogeneous and their tension strength is considerably lesser than compressive one. The structure heterogeneity means the presence of so called «structure defects» inducing the great stresses concentration inside elements under loading. Herewith the main attention is drawn to the tensile stresses concentration connected with so called «rupture failure mechanism» that is fully right in the Fracture Mechanics domain where the tensile average stress σ takes place. But in the pseudoplasticity domain the average stress σ is compressive and the structure defects must induce the other failure mechanism unlike the one under tensile σ. Nevertheless the controlling point of view ascribes also the pure rupture failure to compressed concrete also because the process of rupture microcracks development is observed when the stress level exceeds the so called «level of microcracking R^0_{crc}» [17]. Apparently the microcracks development leads to concrete volume increase, i.e. to the dilatation which is pseudoplasticity characteristic feature. Besides the microcracks are oriented on the first principal stress σ_1 areas owing to what the concrete acquires the anisotropic properties.

The anisotropic materials is differed with the specific constitutive relations $\sigma_{ij} - \varepsilon_{ij}$ from the classical isotropic structural materials. By the anisotropy the shear strains depend on not only tangential stresses but normal stresses also. Accordingly the axial strains are depended on both the normal stresses and tangential ones. The noted «mixed» joint influence of the normal and tangential stresses on the strains means the presence of joint shear-rupture failure mechanism than pure rupture one with the pseudoplasticity. This more realistic notion is confirmed by the clear-cut mixed shear-rupture cracks in failure state of concrete compressed prisms and cylinders (Fig. 1, a). The model of so called «Zigzag-Cracks» explains the mixed failure mechanism under the compression of concrete [18]. Moreover under non-uniform stress-strain states concrete displays the important peculiarities: descending branch of relations $\sigma_{ij} - \varepsilon_{ij}$ induced by the so called «natural strains control», particular redistribution of stresses and the ESC.

STRESS-STRAIN STATE DEVELOPMENT IN CONCRETE NON-UNIFORMLY COMPRESSED ZONE OF RC ELEMENTS

The pseudoplasticity characteristic phenomena can be revealed on the simple example of normal section compression zone of bending

RC element under gradual loading. Herewith the strains and stresses of normal section are changed and successive replacement one after another state and stress-strain distribution happen (Fig. 2, a).

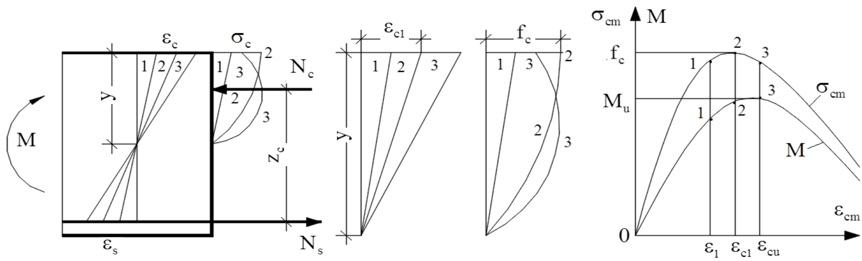

Figure 2. (a) normal section stress-strain state development, (b) strains and (c) stresses in separate concrete compressed zone for states 1, 2, 3 during load increase, (d) conformity of the states 1, 2, 3 on the curves σ_{cm} – ε_{cm} and M – ε_{cm}, σ_{cm}, ε_{cm} – stress and strain respectively on the concrete compressed extreme fibers; the height y changing of concrete compressed zone is not shown conditionally.

Under non-uniform stress-strain states the stresses distribution is governed by certain condition of joint deformations by which may be the known hypothesis of plane section (Fig. 2, a, b). The low enough load levels lead to the elastic almost state 1 (Fig. 2). The high enough load levels induce the curvature stresses distribution and reaching at the concrete extreme fibers the peak stress f_c with the strain ε_{c1} of the concrete compression diagram (state 2 in Fig. 2). Apparently the state 2 can't be the failure because the strains of concrete extreme fibers ε_{cm} obey to the common for all concrete fibers law of plain section and the strains ε_{cm} can't be accelerated arbitrarily. Thus the conditions of joint deformations at the concrete under non-uniform stress-strain states can secure the strains control which is «natural strains control» unlike «artificial strains control» of concrete specimens tested in the special installations. Thanks to the «natural strains control» in the concrete compressed zone after state 2 can be displayed the descending branch of the concrete compression diagram (state 3 in Fig. 2). Herewith the decrease of stresses near concrete extreme fibers is accompanied by the stresses increase near zero line of concrete compressed zone, i.e. the particular stresses redistribution along the height y of compressed zone happens.

It is important to emphasize that noted peculiar phenomena under pseudoplastic state are connected with two competitive processes taking place simultaneously in the concrete compression zone: disstrengthening near extreme fibers and strengthening near zero line. Under loading after state 2 the concrete strengthening prevails firstly over disstrengthening and resultant force N_c in concrete compressed zone increases whereas the arm of inner couple z_c (Fig. 2, a) is not decreased. As a result the section bearing moment M is somewhat enhanced. But increasing concrete disstrengthening brakes the force N_c growth and section moment M reaches the strict maximum M_u, after what the being loaded section can be disstrengthened only. Apparently the maximum moment M_u is ultimate moment of the normal section.

EXTREME STRENGTH CRITERION

The described above phenomena lead to the important conformity of relations between «stress σ_{cm} – strain ε_{cm}» for concrete compressed extreme fibers and «section moment M – strain ε_{cm}» which reflect the essence of pseudoplastic state. The first relation is concrete compression diagram, the second one may be derived on the basis of the continuum Mechanics Equations. The pointed out relations have the same argument ε_{cm} by means of which the section ultimate state reaching can be controlled. Therefore variable ε_{cm} is so called «characteristic deformation». Owing to the same argument ε_{cm} the conformity of the curves $\sigma_{cm} - \varepsilon_{cm}$ and $M - \varepsilon_{cm}$ is displayed as conformity of the states 1, 2, 3 (Fig. 2, d). The substantial peculiarity of relation $M - \varepsilon_{cm}$ is (unlike the perfect plasticity) the presence of strict maximum expressing the Extreme Strength Criterion (ESC)

$$M\left(\varepsilon_{cm}\right)\Big|_{\varepsilon_{cm}=\varepsilon_{cu}} = \max , \qquad (5)$$

where ε_{cu} is ultimate strain of normal section concrete, which can be found from the Continuum Mechanics Equations together with the ESC (5).

GENERAL STRENGTH DESIGN METHOD OF NORMAL SECTIONS (GSDMNS)

The design model, including two design cases, was put into basis of the GSDMNS (Fig. 3).

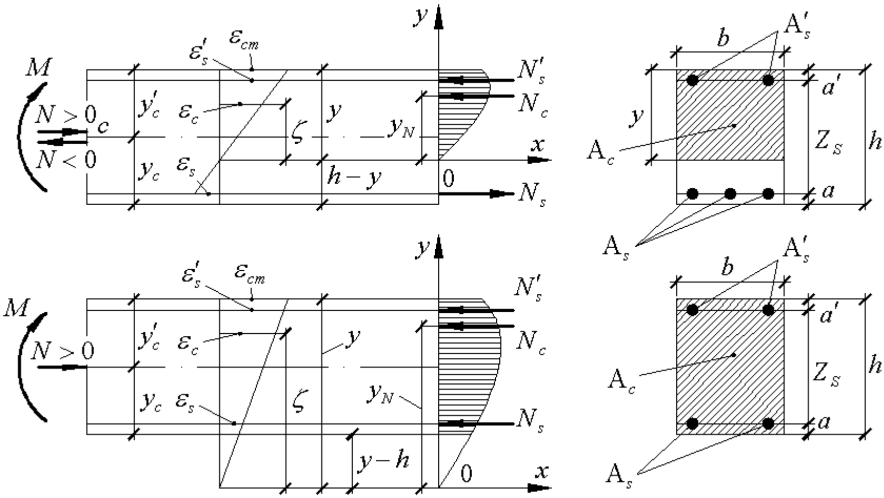

Figure 3. Two cases of strength design of RC elements normal section

The balance equations and corresponding to the law of plane section geometric ratios, written separately for both design cases, together with constitutive relations for concrete and steel form the complete design equations system of the GSDMNS, considered in detail in the [9]. The concrete compression diagram was adopted according to the [1, 2]. The steel tension diagrams were approximated in considerable detail for diagrams with yielding plateau and with yield limit.

For design relations generality the load parameter F as unified external force factor is used by means of which section bending moment M and axial force F are expressed so

$$\mathrm{M} = \mathrm{F f_M}, \qquad \mathrm{N} = \mathrm{F f_N}, \tag{6}$$

where f_M, f_N – moment M and force N intensity respectively which are known usually. Apparently F = M or F = N can be. As a result the ESC must be written in general form

$$\mathrm{F}\left(\varepsilon_{cm}\right)\Big|_{\varepsilon_{cm}=\varepsilon_{cu}} = \max. \tag{7}$$

The Summary Equation System (SES) of the GSDMNS includes two balance equations after removal from the ones the unknown stresses σ_s, σ'_s and strains ε_s, ε_s' of the tensile and compressed reinforcements with areas A_s, A'_s respectively. Herewith the steels tension diagrams and law of plane section are used. As a result the SES is relations between values F, A_s, A'_s, ε_{cm} and concrete compressed zone height y.

TWO MAIN DESIGN PROBLEMS AND THEIR SOLVING

The problems of section strength control and selection of the needed reinforcement are set and solved as problems of non-linear optimization with constraints. The first problem with unknown F, εcm, y is solved with aim function as the ESC (7) and the SES as additional constraints. The second problem with unknown As, Ás, εcm, y is so called «dual problem» [19] relatively first one. This problem aim function is

$$A = A_s + A'_s = \min, \tag{8}$$

with constraints as the SES and additional inequalities

$$\sigma_s \le R_{sd}\,, \quad \sigma'_s \le R'_{sd}\,, \tag{9}$$

where R_{sd}, R'_{sd} are design resistances of tensile and compressed reinforcement respectively.

Design algorithms of the above problems are worked out in considerable detail in the [20]. The design algorithms are simple enough that practical problems can be solved by means of the Table Processor MS Excel.

THEORETICAL AND TEST VALUES PROXIMITY

This question is considered in some detail in the [9], where the tests of bending and eccentrically compressed RC elements are analyzed. Besides that experimental investigations of eccentrically compressed columns of high strength concrete were carried out [21]. During tests the being controlled values were ultimate load, concrete ultimate strain ε_{cu}, reinforcement strains ε_s, $\acute{\varepsilon}_s$ and concrete compressed zone height y. The proximity of pointed out theoretical and experimental values was satisfactory although the variability of ε_{cu}, ε_s, $\acute{\varepsilon}_s$, y values was more than ultimate load. As averaged values it can point out the mean ratio F^{test}/F^{calc} of experimental ultimate load parameter F^{test} to the theoretical one F^{calc} equal 1.06 and the coefficient of variation v = 8 %.

EXAMPLES OF DESIGN

It is considered the strength control problem of rectangular cross section. The aim functions (7) and additional restriction-equality are obtained from the balanced equations (Fig. 3):

for case $y \le h$

$$F=\{\sigma_s'A_s'z_s + f_c by\varphi[h_0 - y\,(1- \psi / \varphi)]\}/[f_M + f_N(y_c - a)], \qquad (10)$$

$$(f_c by\varphi - \sigma_s A_s + \sigma_s'A_s')[f_M + f_N\,(y_c - a)] - \{\sigma_s'A_s'z_s + f_c by\varphi[h_0 - y\,(1 - \psi/\varphi)]\}\, f_N = 0, \quad (11)$$

where quantities φ, ψ are depended on the values $\alpha = \varepsilon_{cm} / \varepsilon_{c1}$ and $K = 1.1E_c\varepsilon_{c1} / f_c$,

for case $y \ge h$

$$F = [\sigma_s'A_s'z_s + f_c bh^2(\underline{\psi} - (y - h_0)\, \underline{\varphi} / h)] / [f_M + f_N\,(y_c - a)], \qquad (12)$$

$$(f_c bh\underline{\varphi} + \sigma_s A_s + \sigma_s'A_s')[f_M + f_N\,(y_c - a)] - [\sigma_s'A_s'z_s + f_c bh^2\,(\underline{\psi} - (y - h_0)\, \underline{\varphi}/h)]\, f_N = 0, \quad (13)$$

where quantities $\underline{\varphi}$, $\underline{\psi}$ are depended on the values α, y, K.

In (10) – (13) the stresses σ_s, σ_s' are expressed by way of α and y, using the steel tension diagrams $\sigma_s(\varepsilon_s)$, $\sigma_s'(\varepsilon_s')$ and geometric relations

$$\varepsilon_s = \pm\varepsilon_{c1}\,(h_0 / y - 1)\alpha, \qquad \varepsilon_s' = \varepsilon_{c1}\,(1 - a' / y)\alpha. \qquad (14)$$

The steel tension diagrams are approximated by the exact enough three-link piece – continuous functions with sections OA, AB, BC (Fig. 4).

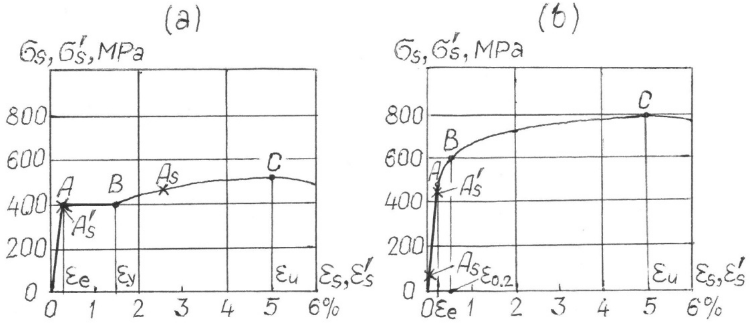

Figure 4. Tension diagram of steel with yielding plateau for Example 1 (a) and with conditional yield strength for Example 2 (b)

As a result the (10) – (13) contain the unknown values α and y only and the first one can be found from the condition

$$F(\alpha) = \max, \qquad (15)$$

where the value y is eliminated by means of additional restriction – equality. Preliminarily the design case $y \le h$ or $y \ge h$ is adopted which is controlled and corrected during calculation process.

The stated algorithm is effected most simply by way of obtaining with numerical step method the curve "Load parameter F – variable α". Herewith the values α are set in succession with small enough step and proper quantities F are calculated up to maximum which determines the ultimate load F_u and corresponding ultimate values α, y, ε_{cm}, σ_{cm}, ε_s, σ_s, $\varepsilon_s{}'$, $\sigma_s{}'$.

EXAMPLE 1: It is given bending element cross section (Fig. 3): $b = 30$ cm, $h = 60$ cm, $a = 5$ cm, $a' = 4$ cm, $h_0 = 55$ cm, $z_s = 51$ cm, $y_c = 30$ cm, $A_s = 4Ø25 = 19.64\ cm^2$, $A_s{}' = 2Ø25 = 9.82\ cm^2$. Steel of A_s, $A_s{}'$ reinforcement is the same and has (Fig. 4, a): $f_{sy} = 400$ MPa, $f_{su} = 500$ MPa, $E_s = 200$ GPa, $\varepsilon_e = 2$ ‰, $\varepsilon_y = 15$ ‰, $\varepsilon_u = 50$ ‰. Concrete normal: $f_c = 30$ MPa, $\varepsilon_{c1} = 2$ ‰, $E_c = 31$ GPa, $K = 2.273$.

It was obtained (Fig. 5): $\alpha = 2.1$, $y = 7.7$ cm, $\varepsilon_{cm} = 4.2$ ‰, $\sigma_{cm} = 7$ MPa, $\varepsilon_s = 25.7$ ‰, $\sigma_s = 452$MPa, $\varepsilon_s{}' = 2$ ‰, $\sigma_s{}' = 400$ MPa, $M_u = 453.2$ KNm.

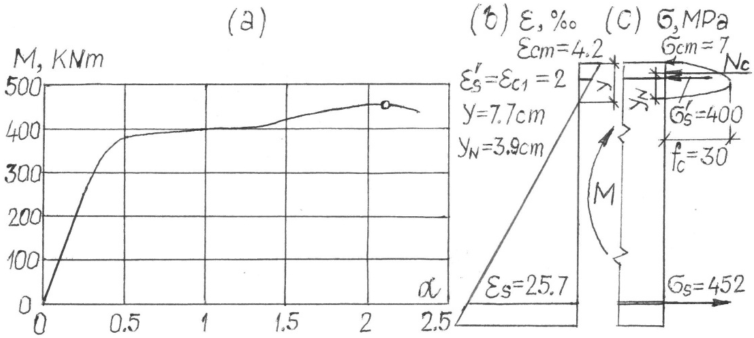

Figure 5. Example 1 design data: (a) curve M – α, (b) strains and (c) stresses distribution in failure state of cross section

The curve "Bending moment M – value α" (Fig. 5, a) shows the considerable enough plastic behavior of cross section in failure state that is induced by the plasticity properties of the used steel with yielding plateau. This result is also connected with comparatively not high the tensile reinforcement ratio $\rho_s = 1.19$ % and not small compressive one ratio $\rho_s{}' = 0.6\,\%$ which retards the brittle failure of concrete. Therefore curve M – α (Fig. 5, a) is very similar to the steel tension diagram in the Fig. 4,a where the states A_s and $A_s{}'$ reinforcements at the failure moment are marked by the crosses. It is seen in the A_s reinforcement state reached the strengthening region.

EXAMPLE 2: It is given eccentrically compressed element section: $b = h = 40$ cm, $a = a' = 3$ cm, $h_0 = 37$ cm, $z_s = 34$ cm, $y_c = 20$ cm, $A_s = 3\text{Ø}20 = 9.42\ cm^2$, $A_s{}' = 3\text{Ø}16 = 6{,}03\ cm^2$. Steel of A_s, $A_s{}'$ reinforcement is the same and has (Fig. 4, b): $f_{0.2} = 600$ MPa, $f_e = 480$ MPa, $f_{su} = 800$ MPa, $E_s = 190$ GPa, $\varepsilon_e = 2.53$ ‰, $\varepsilon_{0.2} = 5.18$ ‰, $\varepsilon_u = 50$ ‰. Concrete normal: $f_c = 33$ MPa, $\varepsilon_{c1} = 1.91$ ‰, $E_c = 34$ GPa, $K = 2{,}066$. Force F eccentricity $e_0 = 3$ cm.

It was obtained (Fig. 6): $\alpha = 1.4$, $y = 42.4$ cm, $\varepsilon_{cm} = 2.7$ ‰, $\sigma_{cm} = 30.7$ MPa, $\varepsilon_s = 0.34$ ‰, $\sigma_s = 65$ MPa, $\varepsilon_s{}' = 2.5$ ‰, $\sigma_s{}' = 472$ MPa, $F_u = 4\ 772$ KN.

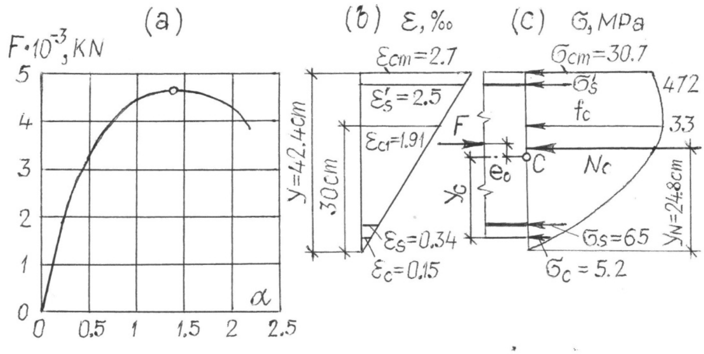

Figure 6. Example 2 design data: (a) curve F – α, (b) strains and (c) stresses distribution in failure state of cross section.

The curve F – α (Fig. 6, a) testifies about lower plastic properties (near 1.5 times) of cross section under eccentrical compression with small eccentricity. The strength of set high-strength steel is used highly insufficiently.

Thus the proposed GSDMNS gives in considerable detail information of stress – strain state of RC elements cross sections under different load conditions. Moreover only the GSDMNS permits to analyze the cross section deformation behavior under ultimate state development and to obtain the ductility indices.

The comparison of ultimate loads obtained according to the GSDMNS and Eurocode design methods [1, 2] showed that latter understates the ultimate moment and overstates the ultimate compressive force near 15 %. The GSDMNS as combining the profound informativity and sufficient accuracy must attract the attention of researchers and designers.

CONCLUSIONS

The concrete under non-uniform compression stays in pseudoplastic state and displays the disstrengthening near most deformed part being restrained by the natural strains control, particular stresses redistribution and as result the ESC. Therefore ESC is natural strength criterion of RC elements normal sections unlike the DSC which does not answer the reality and must give up its place to the ESC.

The ESC together with complete set of Continuum Mechanics Equations leads to the General Strength Design Method of Normal Sections (GSDMNS) which does not demand any additional empirical relationships. The being found from GSDMNS ultimate strain of concrete compressed extreme fibers in normal section ε_{cu} turn out depended on not only the concrete properties but all totality of section conditions too: reinforcement tension diagram and its quantity, section shape, character of load etc. The GSDMNS gives the complete information about stress-strain state of whole normal section including all reinforcement and concrete.

The designs according to the GSDMNS can be realized by means of simple enough software using the known Table Processor MS Excel.

The experiments confirm the fully satisfactory conformity of theoretical and test ultimate load and other values.

The GSDMNS mastering leads to the rise of scientific and qualifying level of designers and students.

The GSDMNS answers the contemporary sciences development trend to general ideas, methods, theories which help to overcome the difficulties connected with huge information stream («information outburst») induced by great complication of concrete and RC properties and corresponding necessity constant increment of experimental researches. When general theories are absent the empiricism prevails and it is impossible to secure the due reliability and optimality of structures because the empirical designs are able to take into account only the factors restricted by the conditions of experiment. On the contrary the general theories show the internal unity of wide section phenomena and lead to considerably more precise designs.

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Received: 23.11.2018