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# SOLVING THE SPATIAL CONTACT PROBLEM FOR THE HINGE JOINTS OF THE BEAM SUPPORT BY THE ELASTIC QUARTER-SPACE AND ONE EIGHTH OF THE ELASTIC SPACE



## ABSTRACT

*The article discusses the solution of the spatial contact problem arising when calculating a reinforced concrete rafter beam pivotally supported by concrete walls. The walls are modeled by the elastic quarter-space on the left and by one-eighth of the elastic space on the right. This contact problem is solved using the numerical method - the Zhemochkin method. For this purpose, the contact area is divided into fragments. Rigid one-way ties are set in the center of each fragment to implement contact between the beam and the wall. It is assumed that the forces arising in these ties provide uniform distribution of reactive pressures in the appropriate fragment. Then, the system of linear algebraic equations for the mixed method of structural mechanics shall be prepared and solved. Different Green functions are assumed for the left and right wall.*

*The problem under consideration is nonlinear, and it requires an iterative process to calculate the effective area of contact and the values of the related reactive pressures. The iterative process shall be finished when contact stresses at the boundary of separation of the structure from* *the walls are identically equal to zero, or when there are no stretched Zhemochkin ties.*

*Isolines of contact stresses and vertical displacements of the contact areas of the walls are plotted for the flexibility index corresponding to the real ratio of rigidity of supported structures and the flexibility index corresponding to the support of the absolutely rigid beam. The function is found, describing the torque arising in the beam versus the distance from the edge of one eighth of the elastic space. A beam can be considered as supported on the left and right by the elastic quarter-space when the distance from the beam axis and the edge of one-eighth of the space exceeds the twofold beam width.*

**Keywords:** flexibility index, Zhemochkin method, contact stresses, Green function, hinged beam.

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# РЕШЕНИЕ ПРОСТРАНСТВЕННОЙ КОНТАКТНОЙ ЗАДАЧИ ШАРНИРНЫХ УЗЛОВ ОПИРАНИЯ БАЛКИ НА УПРУГИЕ ЧЕТВЕРТЬПРОСТРАНСТВО И ОДНУ ВОСЬМУЮ ПРОСТРАНСТВА

## АННОТАЦИЯ

*В статье рассматривается решение пространственной контактной задачи, возникающей при расчете железобетонной стропильной балки, шарнирно опираемой на бетонные стены. Стены моделируются слева упругим четвертьпространством и справа – одной восьмой пространства. Данная контактная задача решается с использованием численного метода – метода Б. Н. Жемочкина. Для этого область контакта разбивается на участки. В центрах каждого участка устанавливаются жесткие односторонние связи, через которые осуществляется контакт балки со стеной. При этом предполагается, что усилия, возникающие в установленных связях, вызывают равномерное распределение реактивных давлений в соответствующем участке. Далее составляется и решается система линейных алгебраических уравнений смешанного метода строительной механики. Для левой и правой стен принимаются различные функции Грина.*

*Рассматриваемая задача является нелинейной и требует итерационного процесса для определения фактической области контакта с величинами соответствующих реактивных давлений. Моментом окончания итерационного процесса служит тождественное равенство нулю контактных напряжений на границе отрыва конструкции от стен либо отсутствие растянутых связей Б. Н. Жемочкина.*

*Построены изолинии контактных напряжений и вертикальных перемещений контактных областей стен при показателе гибкости, соответствующем реальному соотношению жесткостей опираемых конструкций, и показателе гибкости, соответствующем опиранию абсолютно жесткой балки. Установлена зависимость возникающего крутящего момента в балке от расстояния до края одной восьмой упругого пространства. Балку можно считать как опираемую слева и справа на упругое четвертьпространство, когда расстояние от оси балки и края одной восьмой пространства превышает двойную ширину балки.*

**Ключевые слова:** показатель гибкости, метод Б. Н. Жемочкина, контактные напряжения, функция Грина, шарнирно-опертая балка.

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## INTRODUCTION

With the development of foundation construction, the development of methods for calculation of structures on an elastic base was also in progress. Usually, this class of problems in considered as contact problems. Therefore, methods applied in calculation of foundations are also used for solving the contact problems arising in construction tasks for the purposes of design of junction joints for various structures [1].

Lots of models of the elastic base exist. Applicability of each model depends on the nature of the engineering problem. However, many problems are analysed with the combination of various base models. In this area of engineering design, such great scientists are famous as Zhemochkin B.N., Sinitsyn A.P., Shtaerman I.Ya, Korenev B.G., Klepikov S.N., Gorbunov-Posadov M.I., Solomin V.I., Aleksandrov V.M., Bosakov S.V. and others [1–9].

Contact problems are reduced to solving the integral equations, with their solution depending on the integral equation kernel. Only quite limited range of problems has a solution with relatively small amount of mathematical procedures. Therefore, it's unreasonable to solve each contact problem using integral equations. Due to this, B.N. Zhemochkin method is often used successfully for practical purposes [1, 2, 4, 5].

During the design of buildings and facilities, special attention is paid to the junctions for various structures. Hinge joints for beams supported by walls may be considered as examples of such junctions. The design span of a beam and the maximum bending moment depend on the sizes of the effective contact zone in the joint. The precision of calculation of these values is a factor influencing the engineering design of matched structural elements.

It is assumed further that a beam is supported by the elastic quarter-space and one-eighth of the space. Green functions for these elastic bases are taken in accordance with [10].

With this problem formulation, the torque arises along the beam length. For reinforced concrete elements, their twisting resistance is known to be much less than their bending resistance. Also, reinforced concrete elements can break down at early stages of loading. Therefore, with non-symmetric supporting of beams, arising torques must not be neglected [11].

## PROBLEM FORMULATION

The beam under consideration is pivotally supported by the elastic quarter-space on the right and by one-eighth of the space on the left (see Figure 1). The calculation tasks are as follows: to define the contact area; to plot the surfaces of contact stresses and vertical displacements with the real flexibility index and the flexibility index corresponding to the support of an absolutely rigid beam by walls; to find the torque arising in the beam due to the non-symmetric distribution of contact stresses in the beam joints. For these purposes, the following assumptions are accepted:

- bending hypotheses [12] are valid for the beam;
- the wall on the right is modelled as an elastic quarter-space and on the left as one-eighth of an elastic space;
- B.N. Zhemochkin ties are assumed to be one-way, being in compression only;
- shear stresses in the zone of contact are disregarded.

The beam is depicted by its longitudinal axis on the design diagram. As for the places of contact with walls, the beam is replaced there with middle planes *ABCD* and *KFGH*, with their rigidity assumed to be infinite in the direction of the *Y* axis. The B. N. Zhemochkin method [4, 5] is used for calculation. For the detailed description of B. N. Zhemochkin method, as it is applied to the spatial contact problem under consideration, see the author's article [13] where the beam is considered pivotally supported on the left and right by walls as an elastic quarter-space.

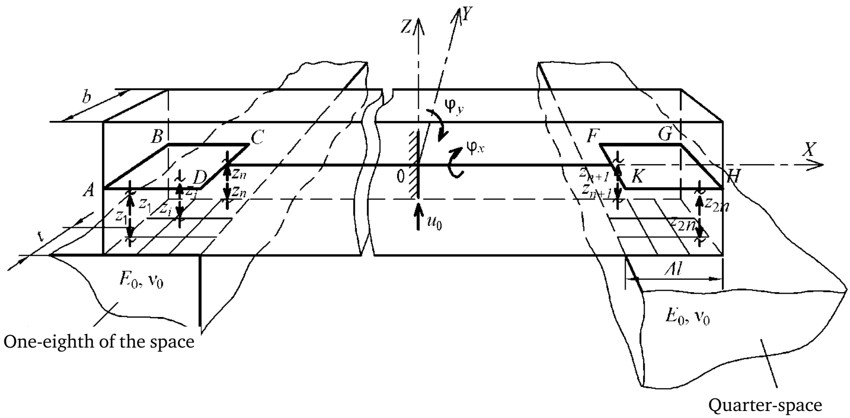


**Figure 1.** Beam design diagram: *ABCD* and *KFGH* are the elements of the beam middle plane; *X0Y* is the coordinate system, where the origin coincides with the middle of the beam span, the *X* axis runs along the longitudinal axis of the beam; the *Y* axis is the horizontal axis; the *Z* axis is the vertical axis; *b* is the beam width; $\Delta l$ is the beam supporting depth; $E_0$, $\nu_0$ are the modulus of deformation and the Poisson ratio of bases; $u_0$, $\varphi_x$, $\varphi_y$ are the vertical and angular displacements of the middle of the beam about the axis *X* and *Y* respectively; $z_i$ is the force in the B.N. Zhemochkin tie; $n$ is the number of B. N. Zhemochkin fragments in a single joint

The design diagram is used to prepare the system of linear algebraic equations for the mixed method of structural mechanics [14]:

$$\begin{cases} \delta_{1,1}z_1 + \dots + \delta_{1,n}z_n + u_0 + \varphi_x \cdot y_1 + \varphi_y \cdot x_1 + \Delta_{1,F} = 0 \\ \dots \\ \delta_{n,1}z_1 + \dots + \delta_{n,n}z_n + u_0 + \varphi_x \cdot y_n + \varphi_y \cdot x_n + \Delta_{n,F} = 0 \\ \delta_{n+1,n+1}z_{n+1} + \dots + \delta_{n+1,2n}z_{2n} + u_0 + \varphi_x \cdot y_{n+1} + \varphi_y \cdot x_{n+1} + \Delta_{n+1,F} = 0 \\ \dots \\ \delta_{2n,n+1}z_{n+1} + \dots + \delta_{2n,2n}z_{2n} + u_0 + \varphi_x \cdot y_{2n} + \varphi_y \cdot x_{2n} + \Delta_{2n,F} = 0 \\ -\sum_{i=1}^{2n} z_i + R_F = 0; -\sum_{i=1}^{2n} z_i \cdot y_i + M_{xF} = 0; -\sum_{i=1}^{2n} z_i \cdot x_i + M_{yF} = 0 \end{cases} \tag{1}$$

where $\delta_{i,j}$ is a vertical displacement of the point *i* due to the impact of the vertical unit force applied in the point *j*; $z_i$ is the unknown force in the B. N. Zhemochkin tie *i*; $u_0$, $\varphi_x$, $\varphi_y$ are unknown vertical and angular displacements in the introduced restraint; $(x_i, y_i)$ are the coordinates of the centre of gravity for the B. N. Zhemochkin fragment with the number *i*; $\Delta_{i,F}$ is the displacement of the point *i* due to the impact of the external load *F*; $R$ is the response in the introduced restraint due to the impact of the external load *F*; $M_{xF}$, $M_{yF}$ are the reactive torques about the axes *X* and *Y* due to the impact of the external load *F*, respectively; $n$ is the number of B.N. Zhemochkin fragments in a single joint.

The major difference between solving this problem and that already published in [13] is calculation of coefficients at unknown values of $\delta_{i,j}$ in the system (1).

These coefficients are calculated using the equation (2):

$$\delta_{i,j} = \lambda W_{i,j} + \frac{1-\nu_0^2}{\pi E_0} V_{i,j}, \tag{2}$$

where $\lambda$ is the flexibility index [1], the dimensionless value depending on the ratio of rigidity values for the elastic quarter-space and the supported beam (3); $W_{i,j}$ is the vertical displacement of the point *i* in the middle place of the beam due to the impact of the unit force applied in the point *j* (standard methods of structural mechanics are used to calculate it [14]); $E_0$, $\nu_0$ are the modulus of deformation and the Poisson ratio of the elastic base; $V_{i,j}$ is the vertical displacement of the point *i* on the surface of the elastic base (the quarter-space and one-eighth of the space) due to the impact of the unit force applied in the point *j*.

The flexibility index [1]

$$\lambda = \frac{\pi E_0 b \Delta l^3}{(1-\nu_0^2) EI}, \tag{3}$$

where $b$ is the beam width; $\Delta l$ is the beam support depth; $EI$ is the beam bending rigidity.

The equation described in [13] is used to calculate the displacements of points on the surface of the elastic quarter-space (for the right joint) $V_{i,j}$. The equation (4), derived from the equation for calculation of vertical displacements of the elastic quarter-space surface in accordance with [10], is used to calculate vertical displacements of surface points for one-eighth of the elastic space (the left joint)

$$V_{i,j} = \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} + A_1 + A_2 + A_3 + A_4 + B_1 + B_2 + B_3 + B_4 \qquad (4)$$

$$A_1 = \frac{a_1 m(x_i - x_j + m)}{[(x_i - x_j + 2m)^2 + (y_i - y_j)^2]^{3/2}} + \frac{1 + a_0}{\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j)^2}} \qquad (5)$$

$$A_2 = \frac{a_1 t(y_i - y_j + t)}{[(x_i - x_j)^2 + (y_i - y_j + 2t)^2]^{3/2}} + \frac{1 + a_0}{\sqrt{(x_i - x_j)^2 + (y_i - y_j + 2t)^2}} \qquad (6)$$

$$A_3 = \frac{a_1 m(y_i - y_j + m)}{[(x_i - x_j + 2m)^2 + (y_i - y_j + 2t)^2]^{3/2}} + \frac{1 + a_0}{\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j + 2t)^2}} \qquad (7)$$

$$A_4 = \frac{1}{\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j + 2t)^2}} + \frac{2(1 + a_0)}{\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j + 2t)^2}} \qquad (8)$$

$$\begin{aligned} B_1 = {} & \frac{2a_0}{\pi\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \operatorname{Arctan} \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{2\sqrt{m(x_i - x_j + m)}} + \\ & + \frac{a_1}{\pi}\left( \frac{\sqrt{m(x_i - x_j + m)}}{(x_i - x_j)^2 + (y_i - y_j)^2} - \frac{2m(x_i - x_j + m)}{\left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{3/2}} \times \right. \\ & \left. \times \operatorname{Arctan} \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{2\sqrt{m(x_i - x_j + m)}} \right) \end{aligned} \qquad (9)$$

$$B_2 = \frac{2a_0}{\pi\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \operatorname{Arctan} \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{2\sqrt{t(y_i - y_j + t)}} +$$

$$+ \frac{a_1}{\pi}\left( \frac{\sqrt{t(y_i - y_j + t)}}{(x_i - x_j)^2 + (y_i - y_j)^2} - \frac{2t(y_i - y_j + t)}{\left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{3/2}} \times \right. \tag{10}$$

$$\left. \times \operatorname{Arctan} \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{2\sqrt{t(y_i - y_j + t)}} \right)$$

$$B_3 = \frac{2a_0}{\pi\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j)^2}} \operatorname{Arctan} \frac{\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j)^2}}{2\sqrt{t(y_i - y_j + t)}} +$$

$$+ \frac{a_1}{\pi}\left( \frac{\sqrt{t(y_i - y_j + t)}}{(x_i - x_j + 2m)^2 + (y_i - y_j)^2} - \right. \tag{11}$$

$$\left. - \frac{2t(y_i - y_j + t)}{\left[(x_i - x_j + 2m)^2 + (y_i - y_j)^2\right]^{3/2}} \operatorname{Arctan} \frac{\sqrt{(x_i - x_j + 2m)^2 + (y_i - y_j)^2}}{2\sqrt{t(y_i - y_j + t)}} \right)$$

$$B_4 = \frac{2a_0}{\pi\sqrt{(x_i - x_j)^2 + (y_i - y_j + 2t)^2}} \operatorname{Arctan} \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j + 2t)^2}}{2\sqrt{m(x_i - x_j + m)}} +$$

$$+ \frac{a_1}{\pi}\left( \frac{\sqrt{m(x_i - x_j + m)}}{(x_i - x_j)^2 + (y_i - y_j + 2t)^2} - \right. \tag{12}$$

$$\left. - \frac{2m(x_i - x_j + m)}{\left[(x_i - x_j)^2 + (y_i - y_j + 2t)^2\right]^{3/2}} \operatorname{Arctan} \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j + 2t)^2}}{2\sqrt{m(x_i - x_j + m)}} \right)$$

where, in equations (4)–(12), $(x_i, y_i)$ are the coordinates of the point *i* on the surface of one-eighth of the elastic space where the vertical displacement is calculated; $(x_j, y_j)$ are the coordinates of the point *j* where the unit force $z_j$ is applied; *m* is the distance between the force application point and the rib of one-eighth of the elastic space, parallel to the axis *0y*; *t* is the distance between the force application point and the rib of one-eighth of the elastic space, parallel to the axis *0x*; $a_0 = \frac{4}{\pi^2 - 4}$ [6], $a_1 = 2{,}1$ [10] are the coefficients.

To calculate vertical displacements $V_{i,i}$, where the displacement calculation point is the same as the force application point, the double integral of the function (4) over the B.N. Zhemochkin fragment is precisely calculated.

With the coefficients and absolute terms calculated for the system of linear algebraic equations (1), the matrix method is used to solve this system. As a result, we have a calculated column vector $\vec{z}$, where the first $2n$ components are the forces in B.N. Zhemochkin ties, and three other components are the displacements in the restraint introduced. Then, the current solution shall be analysed. If negative numbers are among the first $2n$ components, it means that the considered beam will separate from the wall. Therefore, stretched ties must be removed from operation. This circumstance defines the iterative procedure. After each iteration, the calculate column vector $\vec{z}$ is calculated. The iterative procedure shall be stopped when there are no negative components $\vec{z}$ in the solution calculated.

To calculate contact stresses in the joints, resulting forces in B. N. Zhemochkin ties are uniformly distributed throughout the related B. N. Zhemochkin fragments. The pattern of reaction pressures is used to access the effective area of contact between the supported beam and the wall.

Because the problem under consideration is non-linear, torques arise in the beam, depending on the amount of contact stresses, the sizes of the contract area and the distance between the beam and the edge of one-eighth of the elastic space, parallel to the longitudinal axis of the beam.

## NUMERICAL SOLUTION OF THE FORMULATED PROBLEM

We use the reinforced concrete beam BSP6.1 described in the series 1.146.2–10/93, Reinforced Concrete Rafter Beams for Roofing the Buildings with 6- and 9-m Spans, Issue 1 (see Figure 2), supported by concrete walls.

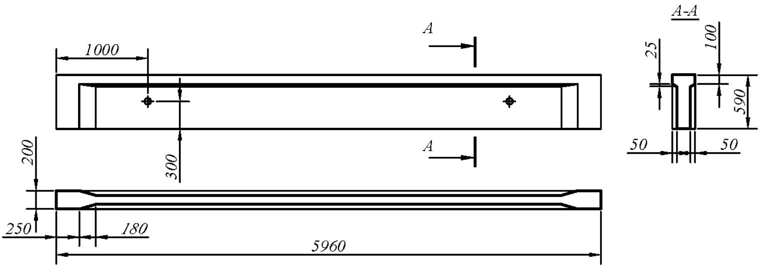


**Figure 2.** BSP6.1 rafter beam drawing

We carry out the calculations for the beam in terms of effects from the concentrated force $F$ applied at the middle of the beam span in the plane $X0Z$. See Table 1 for the initial data for these calculations.

*Table 1*

| Parameter, unit of measurement | Value |
|---|---|
| Beam design length $l$ , m | 5.96 |
| Beam width $b$ , m | 0.2 |
| Beam height $h$ , m | 0.59 |
| Beam support depth $\Delta l$ , m | 0.25 |
| Modulus of elasticity of beam concrete $E$, GPa | 31 |
| Modulus of deformations of wall material $E_0$, GPa | 29 |
| Poisson ratio of wall material $\nu_0$ | 0.19 |
| Concentrated force $F$, kN | 27 |
| Flexibility index $\lambda$ | 2.77 |

The number of B. N. Zhemochkin fragments in the contact zone in the direction of the $X$ axis is assumed to be $n_x$=10, and in the direction of the $Y$ axis, $n_x$=10. The longitudinal vertical edge of the beam is in the same plane as the edge of one-eighth of the elastic space (see Figure 1). With the iterative process taken into consideration, the resulting solution is as follows (see Figure 3).

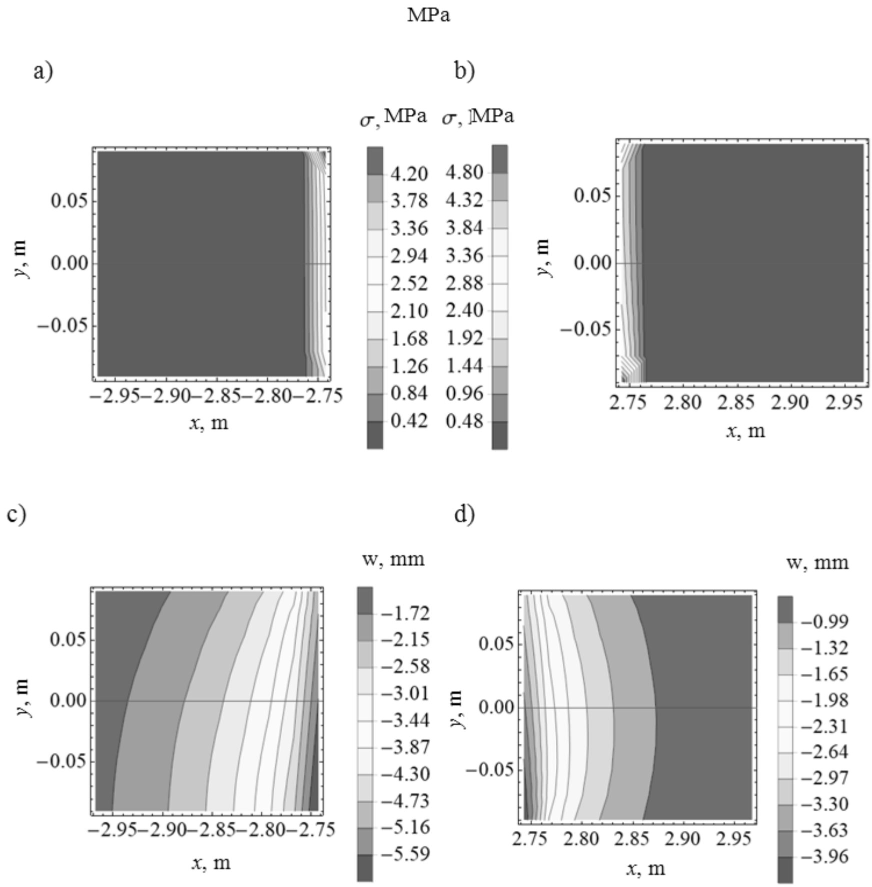


**Figure 3.** Distribution of contact stresses and displacements in hinge joints of a beam: a, b – contact stresses for the left and right joint, respectively; c, d – isolines of vertical displacements of wall surfaces for the left and right joint, respectively

Figures demonstrate that the stress-strain behaviour of joints is different and not symmetric, both about each other and about the longitudinal axis of the beam. Contact stresses for one-eighth of the elastic space are less than those for the quarter-space; however, the reverse is true for displacements. It should be noted that the effective contact area for the left and right joint is reduced to two rows of B.N. Zhemochkin fragments, closest to the ribs parallel with the axis

*OY*. As for the remaining part of the supposed support zone, contact stresses are identically zeros there (see Figure 3a, b, the blue area), corresponding to the beam separation from the wall. Displacements of wall surface points in support zones (see Figure 3c, d), in qualitative terms, correspond to deformations of the elastic quarter-space (the right wall) and one-eighth of the elastic space (the left wall). See Table 2 for comparison of contact stresses and displacements (in absolute values) for the left and right joint.

*Table 2*

| Contact stresses, MPa | | | Displacements, mm | | |
|---|---|---|---|---|---|
| Left joint | Right joint | Difference, % | Left joint | Right joint | Difference, % |
| 4.20 | 4.80 | 14.29 | 1.72 | 0.99 | 42.44 |
| 3.78 | 4.32 | 14.29 | 2.15 | 1.32 | 38.60 |
| 3.36 | 3.84 | 14.29 | 2.58 | 1.65 | 36.05 |
| 2.94 | 3.36 | 14.29 | 3.01 | 1.98 | 34.22 |
| 2.52 | 2.88 | 14.29 | 3.44 | 2.31 | 32.85 |
| 2.10 | 2.40 | 14.29 | 3.87 | 2.64 | 31.78 |
| 1.68 | 1.92 | 14.29 | 4.30 | 2.97 | 30.93 |
| 1.26 | 1.44 | 14.29 | 4.73 | 3.30 | 30.23 |
| 0.84 | 0.96 | 14.29 | 5.16 | 3.63 | 29.65 |
| 0.42 | 0.48 | 14.29 | 5.59 | 3.96 | 29.16 |

The difference between contact stresses for the left and right joint is 14.29 % throughout the contact area. The difference between displacements for the left and right joint is due to the different nature of deformation for the elastic quarter-space and one-eighth of the elastic space.

See Figure 4 for the problem solution with the flexibility index $\lambda = 0$, corresponding to the absolutely rigid beam supported by walls.

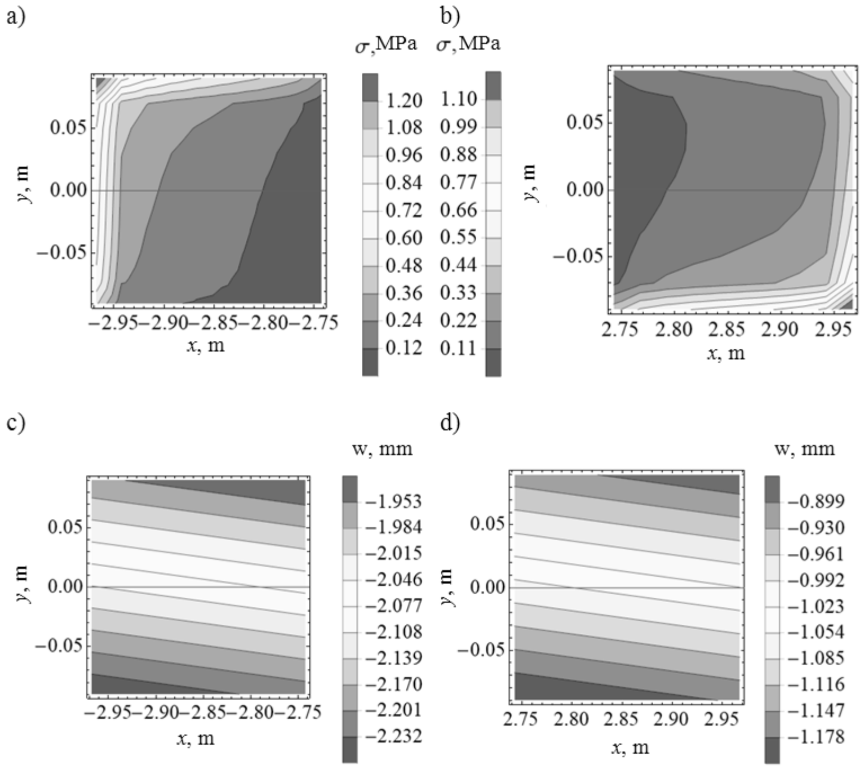


**Figure 4.** Distribution of contact stresses and displacements in hinge joints of an absolutely rigid beam: a, b – contact stresses for the left and right joint, respectively; c, d – isolines of vertical displacements of wall surfaces for the left and right joint, respectively

When the absolutely rigid beam is supported, contact stresses grow nearby the outer boundaries of the contact area; for one-eighth of the space, highest stresses arise in contour points farthest from the edge of one-eighth of the space (see Figure 4a). The flexibility index is $\lambda = 0$; then, as a result of deformation, surface points in one-eighth of the space and, for the equivalent case, in the quarter-space also, are placed not as a surface but as a plane of displacements randomly located in the space (see Figure 4c, d). However, the beam support by the quarter-space on the left and right has resulted only in translational motion of the aforementioned points [13].

Non-symmetric support results in torque arising in the beam; this torque is the same throughout the beam length. See Figure 5 for the plot describing the relative torque versus the relative distance between the longitudinal axis of the beam and the edge of one-eighth of the space parallel to this axis.

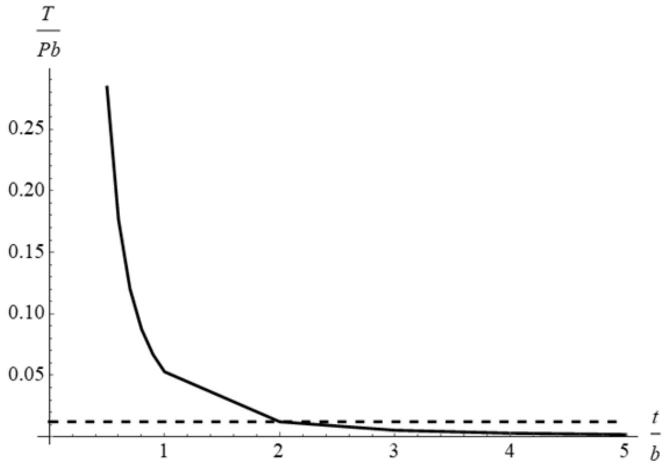


**Figure 5.** Relative torque $\frac{T}{Pb}$ in the beam versus relative distance $\frac{t}{b}$ (see Figure 1) from the edge of one-eighth of the elastic space; *P* is concentrated external force; *b* is beam width

The figure demonstrates that, if the beam is supported at significant distance from the edge of one-eighth of the elastic space, torques fall rapidly and, at the distance $t=2b$ (marked by the dashed line at Figure 5), where $b$ is the beam width, torques become negligible. Here, the pattern of distribution of contact stresses and vertical displacements becomes the same as those for the beam supported by the quarter-space, i.e. the problem formulated here is reduced to the problem in which the beam is supported on the left and right by the elastic quarter-space [13].

## CONCLUSIONS

1. The numerical solution has been obtained for the spatial contact problem where a reinforced concrete beam is supported by one-eighth of the elastic space, at one side, and by the quarter-space, at another side. Contact stresses in one-eighth of the elastic space were found to be less than those for the quarter-space; for displacements, the reverse is true.

2. If an absolutely rigid beam is supported by walls, displacements of points on the wall surface are placed as an inclined plane.

3. It was found that torques arise when a beam is supported non-symmetrically, and these torques are the functions of the distance between the longitudinal axis of the beam and the edge of one-eighth of the elastic space. This function is approximately exponential. At the distance *2b* between the beam and the edge of one-eighth of the elastic space, torques in the beam can be neglected and, therefore, the beam may be calculated as supported by the elastic quarter-space.

4. The demonstrated calculation method is general, and it is applicable to carry out calculations not only for reinforced concrete beams but also for metal and wooden beams, for concentrated forces and other types of loads.

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